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Summary of doctoral thesis

"Spectral gaps, t-designs and ϵ-nets in quantum computing"

Since the introduction of the quantum Turing machine almost half a century ago, quantum computing has emerged as a promising technology and an active interdisciplinary field of research. The quantum circuit model, which is polynomially equivalent to a quantum Turing machine, arose as the dominant formalism in quantum computation. Interestingly, such a gate-based model proved useful not only in describing quantum information processing in quantum computing hardware but also in applications such as many-body quantum systems, quantum complexity, and black hole physics. One of the reasons for such a diverse set of applications is that quantum circuits can be used to model the complexity of quantum states supported on discrete quantum physical systems. The question about the complexity of a specific unitary operation, which prepares the quantum state of interest up to a given precision, is known to be hard. However, the question about the joint upper bounds on such complexities is tangible and can be understood as a question about the computational efficiency of elementary quantum operations used to prepare the states. Indeed, the seminal Solovay-Kitaev (SK) theorem provides such bounds for any discrete universal gate set. However, it is known that certain gate sets enjoy the optimal scaling, which is better than the SK bound.

In this thesis, we explore the bounds on the efficiency of universal gate sets based on the spectral gap of the corresponding t-moment (or averaging) operators. We primarily focus on the finite-scale spectral gaps, which can, in principle, be computed.

Such an approach allows one to derive the non-constructive Solovay-Kitaev-like (SKL) theorems. We demonstrate how to obtain the SKL theorem using a construction based on the correspondence between δ-approximate t-designs and ϵ-nets, which are ubiquitous constructs, widely used in quantum information theory. We achieve such a correspondence by constructing polynomial approximations of the Dirac delta based on heat kernels, which are well-known and natural objects that find many applications in mathematical physics. Using such an approach, we were able to improve the scaling of δ compared to the state of the art, while essentially retaining the scaling of t.

Aside from deriving the mentioned SKL theorem, we provide a relatively simple proof for the poly-logarithmic decay of the spectral gap with calculable constants and an alternative proof for the (global) spectral gap SKL theorem.

Finally, we introduce the notion of the Quantum Circuit Overhead (QCO) and the related notion of *T*-Quantum Circuit Overhead (*T*-QCO), which we believe are suitable measures to compare the efficiency of various gate sets and can be upper-bounded via simple formulas and numerical simulations. Aside from its direct relation to computational efficiency, the notion of the overhead can be used as a reasonable proxy for the actual cost-effectiveness of gate sets

in certain NISQ and fault-tolerant architectures. We use this approach to gain insight into the efficiency of various random ensembles of single-qubit gate sets. Moreover, we numerically analyse several specific choices, such as Clifford+T and Super-Golden gate sets, obtaining interesting results concerning the efficiency of the famous T gate.

Crucially, we put a great emphasis on obtaining formulas where all constants are known or can be calculated in principle via numerical simulations on supercomputing clusters. Such an approach differs from some of the more mathematical works, in which the values of specific constants are not provided.

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