Marco Túlio Quintino
Associate professor (maître de conf.)
Sorbonne Univeristé, CNRS, LIP6
Marco.Quintino@lip6.fr
+33 01 44 27 44 92
https://mtcq.github.io

Paris, 14th of January 2026

Review of the PhD thesis “Spectral gaps, t-designs and ϵ-nets in quantum computing”, written by Oskar Szymon Słowik.

Overall comments:

The main topic of this dissertation is quantum computing, in particular, on understanding the efficiency of universal set of gates. This work aims to have a better understanding of the tradeoff between efficiency and length of composition of a universal set of gates to implement an arbitrary unitary quantum gate. The thesis is clearly written, well-organised, and demonstrates a high level of technical competence. The candidate combines solid theoretical foundations with original research contributions, and places the results appropriately within the context of the existing literature.

The thesis contains 6 chapters, which I now briefly discuss:

Chapter 1: Introduction discusses the quantum circuit model, the concept of universal set of gates, motivates and presents the main question's objective of the thesis, ``to study the calculable bounds on the efficiency of universal quantum gate sets.”

Chapter 2: Preliminaries presents all required mathematical concepts required to understand the novel results and also to follows the mathematical proofs and methods used in the research. This chapter contains 51 pages, and form most part of the new content of the thesis. There, in addition to presenting the mathematical definitions and establishing notation, the author presents an overview of the literature and motivates the problems addressed in this thesis. This chapter is central to unify the novel results of this thesis, and also to ensure the coherence of the thesis.

Chapter 3: Paper I: Calculable lower bounds on the efficiency of universal sets of quantum gates is based on the article written by the author and his supervisor, published in J. Phys. A: Math. Theor. It contains a well-made overview, and a contribution statement.
This article addresses the question of how efficient is a universal set of gates for quantum computing.

As standard, efficiency here is viewed as how the length of the circuit grows with respect to a desired precision for implementing an arbitrary unitary gate. Previous research showed that the efficiency of a universal set S is related to its “spectral gap”, but computing this value is intractable. This work then provides computable methods to obtain the spectral gap in some scale, which is sufficient to bound the efficiency of S in physically feasible cases in which an error is bounded from below. The practicality of their methods is illustrated with numerical experiments.

Chapter 4: Paper II: Fundamental solutions of the heat equation on unitary groups establish an improved relation between ϵ-nets and approximate unitary t-designs is based on the article written by the author, his co-supervisor, and his supervisor, published in J. Phys. A: Math. Theor. It contains a well-made overview, and a contribution statement.
This article addresses the quantitative question of how δ-approximate t-designs form ϵ-nets. While the relationship between approximate designs and ϵ-nets feels seems clear, their quantitative relation is recent, and rigorously established only in Ref.[41], published in 2022. In this article, the authors improve the bounds obtained by Ref.[41], by refining the methods of [41]. The article presented in this chapter finishes with a discussion on efficiency of quantum gates, which leads to the idea of Quantum Circuit Overhead, the central topic of chapter 5.

Chapter 5: Paper III: Quantum Circuit Overhead is based on the article written by the author, his supervisor, and another researcher. This article is still not published in a peer-reviewed journal, but already available as a pre-print at arXiv. The chapter also contains a well-made overview, and a contribution statement.
In this article, the authors introduce and study the relative measure of the efficiency of universal gate sets, which take into consideration the cardinality of the universal set. These definitions should be useful to guide technology to a good choice of universal set of gates. And the authors analyse their definitions with numerical experiments, what makes the results more concrete. A relevant contribution of this article is to present concrete arguments on why the famous T gate is possibly a highly non-optimal choice for the completion of the Clifford gate set, with a performance that is considerably worse than a random unitary, or the so-called super-golden-gate.

Chapter 6 Summary and future directions finishes the thesis with a brief summary of their results, perspectives from the author, and future directions.
In addition to the articles I, II, and III discussed in this dissertation, during his Ph.D., the author had other 8 publications that were not covered by this dissertation. These other publications include scientific articles published in respectable peer reviewed journals, and some publications without the author’s supervisor or co-supervisor.

In summary, this is a well-written, interesting, and highly relevant dissertation. Based on the text, I believe the candidate acquired a good level of scientific maturity and definitely deserves the title of doctor. While I have provided minor comments and suggestions for the author to consider, I strongly recommend its acceptance as a Ph.D. thesis without reservation.

Evaluation:

1 - An assessment, with justification, of whether the dissertation presents the general theoretical knowledge of an applicant for the award of a doctoral degree in a given discipline or disciplines.

The dissertation demonstrates that the candidate possesses a solid and deep comprehension of the general theoretical knowledge required for the award of a doctoral degree in quantum information and quantum computing. The preliminaries and background chapters are clearly written and carefully structured, covering the required concepts of quantum computing, quantum circuits, and the mathematical tools used in the methods. The mathematical concepts are introduced in a compact and direct manner, but with clarity, and a rich discussion on the related literature.

Another merit of this dissertation is to group the topics and mathematical methods of three different research articles in a clear and coherent manner, what makes the text interesting to read. Also, the author conducted numerical experiments to illustrate some of the main results and to have a better view on the practical impact of the mathematical theorems. The ability to use computational methods illustrated the range of the theoretical knowledge of the applicant. Lastly, the level of exposition is definitely appropriate for doctoral level work.

2 - An assessment and justification of whether the doctoral thesis demonstrates the ability of the applicant to carry out scientific work independently for the award of the degree of doctor.

The doctoral thesis clearly demonstrates that the candidate possesses the ability to carry out scientific research independently at the level required for the award of the degree of doctor. The original results are formulated in a clear, precise, and rigorous way. And the discussions and impact are placed within the appropriate context, and analysed and compared with the existing literature. The candidate shows independence in identifying relevant problems, selecting suitable methods, and developing original approaches to address them. The structure of the thesis, coherence of the arguments, and the depth of the technical analysis indicate a high degree of scientific maturity and autonomy in research.

The ability of carrying independent scientific work is illustrated in subsection 2.4.5Research problems, where the author formulates main research problems related to the Thesis, which include questions that were, and were not addressed in his work, opening the door for future directions, which may be taken from the PhD candidate, or another researcher. Also in this direction, Chapter 6, “Summary and future directions” also include a mature and rich summary of the thesis, and interesting future directions.

Finally, while not directly discussed in this thesis, the author had other publications, which includes various different authors, and different topics. Also, there are a few publications without the author’s supervisor, showing a degree of independence, scientific maturity, and theme variety.

3 - An assessment, together with a justification, of whether the doctoral dissertation represents an original solution to a scientific problem, or an original solution to the application of the results of one's own research in the other field.

The doctoral dissertation represents an original and non-trivial contribution to the field, providing original solutions to a relevant and well established questions. The candidate develops new theoretical results that go beyond the existing state of the art and demonstrates the ability to apply advanced methods. The originality of the work is evident both in the formulation of the research questions and in the approaches used to address them. The results obtained are distinguished from previously published work and are properly situated within the relevant literature, highlighting their novelty and significance. The dissertation therefore fulfils the requirement of presenting an original solution to a scientific problem.

Suggestions, questions, and comments:

Chapter 2: Preliminaries

- The relationship between spectral gap and Kazhdan's property (T) is mentioned multiple times in the thesis, but no details about this connection are presented. Maybe it's worth mentioning a bit more on this topic, for instance, explaining what is the Kazhdan's Property (T), and what was the motivation behind this definition.
- The spectral gap of a set S plays a major role in this thesis, but, while this concept is clearly discussed and properly defined, this definition and discussions is scattered in multiple parts of the thesis, and maybe not so easy to spot for readers who are interested in this work, but will not read the whole thesis carefully. Given the major role of spectral gap, I'd suggest the author to present it in an definition environment, as it was done for Def. 2.5 in page 36. I'd use this opportunity to present all equivalent definitions, and the definition of gap(v, t), and the fact that gap(v)=\inf_t gap(v, t). This should help the readers to go directly to the point of knowing what gap(v, t) and gap(v) mean in a clear and direct way.
- As discussed in various parts of this thesis, if a universal set of gates S has spectral gap, then the set S attains optimal asymptotic efficiency, that is, $\ell(S, \varepsilon) = \Theta(\log(1/\varepsilon))$. But, the author does not discuss why this is true, or the intuition behind this result. This thesis presents an alternative proof of this result (which was first presented in Ref.[111]), so, strictly speaking, the author does discuss it. However, I feel that the thesis would benefit from an informal discussion in chapter 2 on why this result is true. And, on why, it is believed that all universal set of gates have spectral gap. Still on this topic, maybe it would be good to address the converse question: If a universal set of gates S attains optimal asymptotic efficiency, does it imply that S has spectral gap? I had the feeling that this approach was not considered, and this view could provide the direction to prove that all universal set of gates have spectral gap.
- As pointed by the author, the standard proof of SK is constructive, in the sense that there is an explicit and simple algorithm to decompose an arbitrary gate into a finite set of universal gates (with any desired precision). However, the SKL theorems based on spectral gap are not constructive. Do we have hopes that one can find an

algorithm to decompose an arbitrary gate that attains optimal asymptotic efficiency? I feel that, from a practical and a foundational perspective, this question is crucial. If we know that an optimal decomposition exists, but we have no idea on how to do it, results on spectral gap would not necessarily imply in benefits from practical applications to quantum compiling. I feel that the thesis could have addressed this question, even if just a single paragraph.

Chapter 3: Paper I: Calculable lower bounds on the efficiency of universal sets of quantum gates

- This work includes numerical results and numerical experiments, which illustrate the main results and help us to understand their possible practical impact. However, the code used to obtain these results is not available anywhere. Moreover, there are not many details on how the numerical experiments were done, the approach, the language which was used, etc. I would say that it is very important and useful to make code used in this work openly available. In addition to clarify and transparency (this would simplify someone to check the validity of the results), future researchers could use these code for analogous problems. Or, to use the code to obtain similar plots, but with different values.
- The main motivation of this chapter is to obtain lower bounds on gap(S), where S is a universal set of gates. However, since this question is intractable, the authors focus on the spectral gap at a certain scale, a problem that is computationally feasible. But, a neutral questions is, why bounding gap_t(S) is useful, from a quantitative and rigorous way, how does information from gap_t(S) helps one to tackle the problem of a universal set of gates to be efficient? Do authors do write that this is useful when one “is interested in a physically feasible case, in which an error ε is bounded from below.”. But, I believe that this connection was not so explicit/clear and could have been detailed and discussed more. Maybe not in the article, but in the overview part of the thesis.

Chapter 4: Paper II: Fundamental solutions of the heat equation on unitary groups establish an improved relation between ϵ-nets and approximate unitary t-designs

- As stated by the authors, an immediate open question is to ask whether the better bound provided in this work is tight. A refined version of this question is, does the author see room for improvement using the same methods, or, small variations of the technique used in Ref.[41] and Paper II ? Or, the author thinks that, in order to improve the new bound (or to prove its optimality), novel and methods would be required.

Chapter 5: Paper III: Quantum Circuit Overhead

- The clearly stated by the authors, the quantity Q(S, ϵ) is an upper bound on the Quantum Circuit Overhead. But, I did not find a discussion of how tight or good this bound is. For instance, could it be the case that some numerical results are “distorted” due to the fact that the QCO may greatly differ from Q(S, ϵ). Would this be a plausible explain on why the super-golden-gate is not the optimal gate to be added for Clifford group to obtain universality?

Minor comments and typographical corrections:

- In page 10, there seems to be a typo in the domain of the indicator function.
- There is a typo in Eq. 2.85
- There is a full stop missing after Eq. 2.73.
- There is a typo after 2.150. “that that”
- While it’s a relatively well-known fact, I feel that the sentence “The Clifford circuits (i.e. circuits made of Clifford gates) are known to be efficiently simulable on classical computers.” after Eq.2.129 requires a citation.
- In page 41, maybe it is useful to define the diamond norm, or at least to provide a bit more details. For instance, to mention that in the diamond norm is the one norm for states, where there is an optimisation regarding possibly entangled states. This could have been mentioned in example 2.7 for instance.
- There seems to be a typo in the last sentence of page 86. The bound on delta presented there is not the one from Thm.2, but the one from Ref.[41]
- NISQ appears multiple times in the thesis, and even in the examples 1 and 2 of paper III. However, the thesis does not really explain what is NISQ, and the relevance of NISQ for their work. I had the feeling that, NISQ references are a bit artificial. Maybe my suggestion would be to remove unnecessary references to NISQ, or to explain a bit better what NISQ is, and why it is an important motivation for this thesis.

Final remarks:

For all the reasons detailed above, I am confident that this dissertation meets the high standards and all the requirements of Ph.D thesis.
Therefore, I conclude that the presented dissertation meets the formal requirements for a Ph.D. thesis and recommend admission of the Candidate to the subsequent states of the procedure, including the public defense.

Paris, 14th of January 2026
Marco Túlio Quintino

Center for Theoretical Physics
Polish Academy of Sciences
Aleja Lotników 32/46, 02-668 Warsaw
Tel. (+48 22) 847 09 20, Fax/Tel: (+48 22) 843 13 69
E-mail: cft@cft.edu.pl, NIP: 525-000-92-81, REGON: 000844815

Konkluzja recenzji rozprawy doktorskiej
(Conclusion of dissertation review)

„Spectral gaps, t-designs and ε-nets in quantum computing"

Tytuł rozprawy *(Dissertation title)*: .....

Oskar Słowik

Autor rozprawy *(Author of the dissertation)*: .....

☒ Pozytywna ocena *(Positive conclusion)*:

Stwierdzam, że przedstawiona mi do recenzji rozprawa spełnia wszystkie wymagania ustawowe i zwyczajowe stawiane rozprawom doktorskim i wnoszę o dopuszczenie jej do dalszych etapów postępowania doktorskiego, uwzględniając publiczną obronę.

*(I conclude that the presented dissertation meets the formal and customary requirements for doctoral dissertations and I recommend its admission to subsequent stages of the procedure, including the public defense.)**

☐ Ocena negatywna *(negative conclusion)*

Stwierdzam, że przedstawiona mi do recenzji rozprawa nie spełnia wszystkich wymagań ustawowych i zwyczajowych stawianych rozprawom doktorskim i dlatego nie rekomenduję dopuszczenia jej do dalszych etapów postępowania doktorskiego.

*(I conclude that the presented dissertation does not meet the formal and customary requirements for doctoral dissertations and therefore I do not recommend its admission to subsequent stages of the doctoral procedure.)**

Uzasadnienie powyższej oceny znajduje się w raporcie będącym załącznikiem 1.
(The justification of the above assessment can be found in the detailed report in the attachment 1.)

14/01/2026
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Data i podpis
(Date and signature)

MTQuintino

Załącznik 1: Recenzja rozprawy doktorskiej
(Attachment 1: Review of the dissertation)

Zaznacz ocenę (Please tick the box with your conclusion)