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Summary of the doctoral thesis

"Super-Tonks-Girardeau effect in the low-dimensional Bose gases with competing interactions"

One of the foundational achievements of quantum mechanics was the discovery that particles fall into two distinct categories - bosons and fermions - each governed by fundamentally different statistical laws. For decades, the properties of these two types of particles have served as a foundation for the development of numerous fields in physics. One of the intriguing outcomes of these investigations was the discovery of a deep similarity between strongly repulsive bosons and non-interacting fermions in one-dimensional systems. In bosonic systems, the presence of strong repulsive interactions effectively prevents two particles from occupying the same position, thereby mimicking the Pauli exclusion principle characteristic of fermions. This formal resemblance leads to agreement in both the energy and the second-order correlation functions of the two systems, forming the basis for the concept of "fermionized" states.

Noteworthy, this similarity arises not only in the case of strongly repulsive bosons, but also in certain highly excited states of systems with strongly attractive interactions. These counterintuitive states are known as super-Tonks-Girardeau states. Remarkably, their existence has been experimentally confirmed by first preparing a fermionized bosonic state, and then suddenly quenching the interaction character from strongly repulsive to strongly attractive. These experiments revealed a surprising stability of such systems, as well as their resistance to thermalization.

This thesis presents the theoretical foundations of this phenomenon and explores its potential application in realizing the so-called pumping process, which enables the controlled generation of highly excited states. Particular attention is given to strongly correlated excited states of an ideal Bose gas, in which the properties of a system exhibit striking similarities to those of non-interacting fermions in models with reduced length. In contrast to "classical" fermionization, however, the observed similarities obey the energy and momentum distribution, while significant differences remain in the second-order correlation functions.

In the second part of this thesis, the consequences of sudden changes in interaction strength are analyzed for bosonic systems featuring two competing types of interactions: strongly repulsive contact interactions and long-range attractive ones. It is shown that self-bound states arising in such systems can be classified into two distinct types based on their behavior after the quench: (i) for sufficiently strong nonlocal attraction, the system remains stable, and among the eigenstates of the post-quench Hamiltonian, a

super-Tonks-Girardeau-like state appears; (ii) there also exist self-bound states that disintegrate after the quench - in which case no super-Tonks-Girardeau state is observed.

Throughout this dissertation, a broad range of research methods is employed, with particular emphasis on analytical many-body methods (including the *Bethe Ansatz*), as well as *ab initio* numerical methods, including modern tensor network techniques and exact diagonalization using the Lanczos algorithm.

The key results of the projects presented here include

- the identification of states of an ideal Bose gas that exhibit the same momentum distribution as non-interacting fermions, which can be generated through a pumping process;

- and the demonstration that some of the bound states in systems with dipolar interactions do not have a counterpart in the form of an analogous super-Tonks-Girardeau state and disintegrate following a sudden change in interactions.

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