Dissertation Report

Title of the PhD thesis:
Genuine multipartite entanglement and nonlocality of quantum stabilizer states

Name of the candidate: **Owidiusz Makuta**

The PhD dissertation presented by Owidiusz Makuta studies entanglement and nonlocality properties of qubit stabiliser subspaces, in particular regarding their strong form called genuinely-multipartite. The candidate explores not only the the traditional sense in which the concept of genuine-multipartite non-classicality is defined, but also the newly-advocated-for local-operation-and-shared-randomness (LOSR) network sense. Besides the foundational interest in this work, the thesis provides results relevant for the study of resources in quantum computation, in particular regarding the so-called graph states. The thesis presents solid original results, which have been disseminated via four different papers, two of which have already been published in high-impact venues. I believe that this PhD thesis constitutes a valuable piece of work for the research community in the field. It is worth noticing that Mr. Makuta is first author in three out of the four papers in this thesis (two of which are already published), and has moreover co-authored other three publications (not included in this PhD dissertation), which attests to the ability of the candidate to carry out scientific work independently. In addition, the research carried out in this PhD dissertation required the development and use of highly technical mathematical tools, which further attests for the candidate's analytic and technical knowledge of the topic.

The dissertation is broadly presented in three parts. The first one is a recap of the basic concepts where the results are framed. The second one pertains to the results regarding traditional notions of genuine-multpartite entanglement (GME) and genuine-multipartite nonlocality (GMNL). The third one is about the results on LOSR-network notions for GME and GMNL. After this, the thesis includes conclusions and outlook.

Let me first comment on the the results discussed in Chapters 3 and 4, pertaining to the traditional notion of GME and GMNL. Chapter 3 presents the results of Ref. [1]. There, the candidate studies entanglement in the qudit stabiliser formalism, in particular how to certify that states are GME. The publication presents a variety of results, including (i) necessary and sufficient conditions for a qudit stabiliser subspace to be GME (one method for arbitrary local dimensions and another method for arbitrary prime local dimensions), (ii) upper bound on the dimension of a GME stabiliser subspace, and (iii) certification of GME stabiliser subspaces in terms of the negative-partial-transposition criteria. Chapter 4 presents the results of Ref. [2]. There the goal is to study the relationship between GME and GMNL for qubit stabiliser subspaces, in the context of the so-called 'Gisin conjecture'. The authors show that in this case

GME $\Rightarrow$ GMNL in its stronger form of multipartite full nonlocality. To prove this the candidate combined technical tools available in the literature. The results of these publications then provide timely progress on the question of how to identify and certify (the allegedly most valuable form of) entanglement of quantum states, especially in the many-body regime relevant for quantum computation.

I will now comment on the results discussed in Chapters 5 and 6, which pertain to the LOSR-network definition of GME and GMNL (termed here LOSR-GME and LOSR-GMNL) that has recently been enthusiastically explored by the research community. Chapter 5 presents the results of Ref. [3]. There, the candidate shows that if one has only access to an LOSR-network with bipartite quantum states and any arbitrary source of shared randomness then one cannot prepare any nontrivial graph state with prime local dimension. That is, one would need some extra resources in the network if one wanted to prepare a graph state that had entanglement depth larger than 2. In order to obtain these results, the candidate had to overcome three mathematical and conceptual issues that required a high level of technical mastery to solve. This paper hence highlights graphs states that have the potential to be powerful resources for quantum information tasks, both in gedanken and in real experiments. Finally, Chapter 6 presents the results of Ref. [4]. There, the candidate first explores various types of graph states and shows that some of them are GMNL, and that some others are LOSR-GMNL. Second, the candidate introduces a new type of nonlocality, which they term Local-Operations-and-Neighbour-Communication GMNL (LONC-GMNL). LONC-GMNL assesses the genuine multipartite non-classicality of the observed statistical data by comparing it to the behaviours that can be obtained by allowing not only network-LOSR operations but also communication between neighbours in the network. The candidate then explores how states may display GMNL in one paradigm but not another, and how the assessment of GMNL in terms of LONC highly depends on the structure of the underpinning network. The results in this paper highlight how more foundational work is needed on the characterisation of non-classicality and genuinely-multipartite non-classicality before one can use these to define nonlocality quantifiers that serve as faithful benchmarks for non-classicality (and so potentially for quantum advantage for communication and information processing tasks). The methodology used in these works creatively combined state of the art tools, including the use of the technically-challenging inflation technique.

All in all, this very well-structured PhD dissertation carefully covers various aspects in the study of the non-classical properties of graph states, with focus on the quantification of their resourcefulness from the viewpoint of different paradigms. Despite of the various strengths of the dissertation which I listed above, I nonetheless found some minor weaknesses, which I comment on an annex attached to this letter. I would like to emphasize, however, that the points I raise there are minor, and that I do consider that this PhD dissertation is an outstanding piece of work at the forefront of research.

Therefore, I conclude that the presented dissertation meets the formal requirements for a Ph.D. thesis and recommend admission of the Candidate to the subsequent stages of the procedure, including the public defense. In addition, I would like to propose this PhD dissertation for the distinction.

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Major comments

The first comment I have pertains to the way the candidate discusses Local correlations in the preliminaries section.

The student defines local correlations in Eq. (2.44), which then interprets as "a measurement outcome cannot have an immediate influence on the outcome of another, spatially separated measurement" (bottom of page 19). Unfortunately, this claim is not an interpretation of Eq. (2.44) but rather a consequence of it. Indeed, one cannot recover Eq. (2.44) merely from the claim between quotation marks. For instance, how would one conclude from 'Alice's output cannot influence Bob's output' that also 'Alice's input cannot influence Bob's output', as in the case in Eq. (2.44)?

I kindly ask the student to fix this. Notice that it is not necessary to include such an intuitive account of local correlations in Sec. 2.3.3, so the candidate may as well remove that part of the text.

The other issue pertains to the motivation for a different notion of GME than the traditional one. I don't consider this a 'major' issue in the same sense as the interpretation of local correlations, but rather as a comment that could require some possibly longer discussion.

In the dissertation the student presents the following example. First, a tripartite system in a Hilbert space $\mathcal{H}_2 \otimes \mathcal{H}_4 \otimes \mathcal{H}_2$, prepared on a state $|\psi_e\rangle$ (defined in the thesis, Eq. (2.67)) which is GME in the traditional sense. Second, a fourpartite system in a Hilbert space $\mathcal{H}_2 \otimes \mathcal{H}_2 \otimes \mathcal{H}_2 \otimes \mathcal{H}_2$, prepared on a state $|\psi'_e\rangle$ (defined in the thesis, Eq. (2.69)) which has entanglement depth 2. The argument then goes as: a simple relabelling of the basis elements of the Hilbert space $\mathcal{H}_4$ allows one to rewrite the state $|\psi_e\rangle$ (which has entanglement depth 3) just as $|\psi'_e\rangle$. The question then is 'how can one simple relabelling make a GME state not GME anymore?'. Namely, how could the assessment of GME be basis dependent? I find the way the student presents the discussion a bit cumbersome; for instance, it opens the door to asking about the necessity of assuming or not a physically-relevant subsystem structure of $\mathcal{H}_4$. I think that looking at the argument the other way around might be clearer: you start with four qubits distributed among three parties and prepared in state $|\psi'_e\rangle$, and then by applying a local coarse-graining you obtain a tripartite system in state $|\psi_e\rangle$. How could a simple local coarsegraining increase the entanglement depth of the state? In principle one would require that the definition of GME be closed under such local operations. Maybe the student could consider adding in the manuscript such a take on the example.

Another motivation for the need for an improved definition of GME was presented in Sec. 4 of Ref. [5]. There, an argument was made in terms of resource theoretic arguments and physical operations. What does the student think of this example, and why did they prefer not to go down this route? What is the strength of the example the candidate chose to present, which makes it more suitable for the preliminaries section of this dissertation? The student may choose to update the manuscript of the dissertation, or simply make some comment during the defense.

Minor comments

Here I present some minor comments and questions on the phrasing of concepts, typos, missing references, and the like.

- Page 3: the candidate says
  "This led to the development of the framework called local operations and shared randomness network, which then allows us to construct new definitions of genuine multipartite entanglement and nonlocality by rejecting all states/behaviors that originate from this network [35], [36]."

One of the seminal papers on LOSR-GME and LOSR-GMNL is Ref. [5], which the candidate seems to not be aware of. I recommend the candidate to include this citation here and in other appropriate parts throughout the thesis and the preprints of Ref. [2, 4].

- Page 9, Sec. 2.2.1 first paragraph: typo, it says
  ".. the fact that is that .."
  Please delete "is that".
- Page 11, before Sec. 2.2.2: what does it mean for a stabiliser subspace to be a code? For completeness I'd suggest to add a small comment/footnote or a reference.
- In general, I suggest the addition of "Ref." and "Eq." when referencing to sections, citations, equations, etc. For example, at the bottom of page 14 it says
  " .. using (2.28)."
  I suggest you write ".. using Eq. (2.28).".
- Page 14, last sentence: typo. It says
  ".. computed using (2.28), in the above basis, .."
  I guess there should be a stop and not a comma after "(2.28)"?
- Page 17, after Eq. (2.38). You say
  "Intuitively, entangled states are those for which the separate description of the two subsystems does not allow us to reconstruct the original state. "
  How do you reconcile this phrasing of entanglement with the concept of 'local tomography', given that quantum theory is locally tomographic?
- Page 19, top: it says
  "This leads us to the local hidden variable model which states that any bipartite probability distribution of two instantaneous measurements can be described as.."
  This phrasing can be misleading, given the ".. can be described .." part. I suggest rephrasing it as something like
  "This leads us to the local hidden variable (LHV) model. A bipartite probability distribution of two instantaneous measurements admits of an LHV model when it can be described as..".
- Page 19, after Eq. (2.44): it says
  ".. (see Ref. for a discussion on this topic [74]) .."
  I guess that the '[74]' is misplaced and should go after 'Ref.'?
- Page 20, paragraph after Eq. (2.48). In various places you say things like 'party B can measure b' when I guess you mean 'party B can obtain measurement outcome b'. This is confusing, because 'party B can measure b' can also be understood as 'the measurement setting of B is b'. If my understanding of your meaning is correct, I suggest you rephrase the 'can measure' as 'can obtain measurement outcome' in the various places.
- Page 26, after Eq. (2.68): it says
  ".. this operation is neither invertible nor does it correspond to any physical operation."
  This change of basis is invertible, so I don't understand in what sense the student says otherwise. In addition, this change of basis is not always uphysical. So I suggest the student to rephrase this sentence along the lines of
  ".. this operation does not need to correspond to a physical operation."
- Page 27, first paragraph: typo. It says "3-paritte" instead of "3-partite'.
- Page 27, second paragraph: typo. It says "M -partie" instead of "M -partite".
- General comment: the student seems to use " as opening quotation marks throughout the thesis rather than ". I suggest the correct latex command is used.
- Page 27, after Eq. (2.71): typo. It says 'Hilber' rather than 'Hilbert'.
- Page 32, last paragraph: typo. It says 'stated generated' when it should be 'state generated'.

- Page 32, last paragraph: typo. It says 'a prior unclear' when it should be 'a priori unclear'.
- Page 33, first paragraph. When first mentioning here the state $|\mathrm{GHZ}_{3,2}\rangle$ I'd remind the reader of its definition. For instance, this could be done by adding '(as defined in Eq. (2.31))'.
- Page 38, second paragraph: I find the second sentence difficult to parse. I suggest the text "different to the one mentioned above" be put between brackets.
- Page 55, last paragraph: typo. It says 'strait forward' and it should be 'straightforward'.

References

[1] O. Makuta, B. Kuzaka, R. Augusiak. *Fully non-positive-partial-transpose genuinely entangled subspaces*. Quantum 7, 915 (2023).

[2] O. Makuta, R. Augusiak. *All genuinely entangled stabilizer subspaces are multipartite fully nonlocal*. arXiv preprint arXiv:2312.08757.

[3] O. Makuta, L. T. Ligthart, R. Augusiak. *No graph state is preparable in quantum networks with bipartite sources and no classical communication*. npj Quantum Inf. 9, 117 (2023).

[4] X. Coiteux-Roy, O. Makuta, F. Curran, R. Augusiak, M.-O. Renou. *The genuinely multipartite nonlocality of graph states is model-dependent*. arXiv preprint arXiv:2404.15861

[5] David Schmid, Thomas C. Fraser, Ravi Kunjwal, Ana Belen Sainz, Elie Wolfe, Robert W. Spekkens. *Understanding the interplay of entanglement and nonlocality: motivating and developing a new branch of entanglement theory*. Quantum 7, 1194 (2023).

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Konkluzja recenzji rozprawy doktorskiej
(Conclusion of dissertation review)

Tytuł rozprawy *(Dissertation title)*: **Genuine multipartite entanglement and nonlocality of quantum stabilizer states**

Autor rozprawy *(Author of the dissertation)*: **Owidiusz Makuta**

☒ Pozytywna ocena *(Positive conclusion)*:

Stwierdzam, że przedstawiona mi do recenzji rozprawa spełnia wszystkie wymagania ustawowe i zwyczajowe stawiane rozprawom doktorskim i wnoszę o dopuszczenie jej do dalszych etapów postępowania doktorskiego, uwzględniając publiczną obronę.

*(I conclude that the presented dissertation meets the formal and customary requirements for doctoral dissertations and I recommend its admission to subsequent stages of the procedure, including the public defense.)**

☐ Ocena negatywna *(negative conclusion)*

Stwierdzam, że przedstawiona mi do recenzji rozprawa nie spełnia wszystkich wymagań ustawowych i zwyczajowych stawianych rozprawom doktorskim i dlatego nie rekomenduję dopuszczenia jej do dalszych etapów postępowania doktorskiego.

*(I conclude that the presented dissertation does not meet the formal and customary requirements for doctoral dissertations and therefore I do not recommend its admission to subsequent stages of the doctoral procedure.)**

Uzasadnienie powyższej oceny znajduje się w raporcie będącym załącznikiem 1.
(The justification of the above assessment can be found in the detailed report in the attachment 1.)

22.08.2024 [signature]
Data i podpis
(Date and signature)

Załącznik 1: Recenzja rozprawy doktorskiej
(Attachment 1: Review of the dissertation)

***Zaznacz ocenę (Please tick the box with your conclusion)**