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Konkluzja recenzji rozprawy doktorskiej
(Conclusion of dissertation review)

Genuine multipartite entanglement and nonlocality of quantum stabilizer states

Tytuł rozprawy *(Dissertation title)*:
………
…

Owidiusz Makuta

Autor rozprawy *(Author of the dissertation)*:
……

■ Pozytywna ocena *(Positive conclusion)*:

Stwierdzam, że przedstawiona mi do recenzji rozprawa spełnia wszystkie wymagania ustawowe i zwyczajowe stawiane rozprawom doktorskim i wnoszę o dopuszczenie jej do dalszych etapów postępowania doktorskiego, uwzględniając publiczną obronę.
*(I conclude that the presented dissertation meets the formal and customary requirements for doctoral dissertations and I recommend its admission to subsequent stages of the procedure, including the public defense.)**

☐ Ocena negatywna *(negative conclusion)*

Stwierdzam, że przedstawiona mi do recenzji rozprawa nie spełnia wszystkich wymagań ustawowych i zwyczajowych stawianych rozprawom doktorskim i dlatego nie rekomenduję dopuszczenia jej do dalszych etapów postępowania doktorskiego.
*(I conclude that the presented dissertation does not meet the formal and customary requirements for doctoral dissertations and therefore I do not recommend its admission to subsequent stages of the doctoral procedure.)**

Uzasadnienie powyższej oceny znajduje się w raporcie będącym załącznikiem 1.
(The justification of the above assessment can be found in the detailed report in the attachment 1.)

01/09/2024
……………………………..
Data i podpis
(Date and signature)

**Zaznacz ocenę (Please tick the box with your conclusion)*

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Załącznik 1: Recenzja rozprawy doktorskiej
(Attachment 1: Review of the dissertation)

This thesis investigates entanglement and non-locality in stabiliser states, a critical resource across quantum information. It makes several considerable new contributions to the field and extends our understanding of entanglement and non-locality in general, and for these specialised states. Furthermore, it introduces many novel and powerful tools that will no doubt be useful beyond the scope of this thesis. Entanglement and non-locality have been increasingly understood as key resources in quantum advantage across computation, communication and metrology. The more we have been able to understand these resources, the more we have been able to exploit them, and harness their key features for advantage in practice. The focus on stabiliser states is also prudent and far reaching, as these states are the key entangled state resources across almost all of quantum information, including error correction (and almost all forms of fault tolerant quantum computing), measurement based quantum computing, quantum networks and quantum sensing. One can expect the results of the thesis to then have both direct consequences, for example in quantum networks, and potentially develop our understanding of the role of these quantum features for different quantum technologies.

The thesis is made up of seven chapters, beginning with an introductory chapter, then a second chapter introducing technical details, then four research chapters presenting published or submitted works, and a final concluding chapter. The thesis is well written. Each of the four research chapters consists of an introductory text, followed by the published or submitted work. These introductions are clear, concise and give a useful link binding the thesis together. The articles themselves are very high quality, published or submitted to leading journals. I will now go through the chapters and finish with a summary and some comments.

The introductory chapters 1 and 2 introduce and cover all the necessary background for the thesis. In chapter one the main questions and motivations behind the thesis are presented, and the plan of the thesis is outlined. Chapter 2 introduces the technical preliminaries necessary to follow the thesis. It begins with a quick introduction to standard quantum information formulation of states and measurements and then moves on to cover the more specialised topics of the thesis: stabiliser states, graph states, entanglement and non-locality (in first the bipartite, then multipartite settings), as well as the inflation technique - a particular form of proof by contradiction that is used in the later chapters extensively. It is well written, and well balanced, clearly explaining complicated and subtle notions, and in my opinion would serve as an excellent reference for students starting out in the field. Furthermore, for the thesis itself, it provides a good balance of covering all the necessary information and not being too overbearing. The examples are useful and I found it helped a lot to follow the technical research chapters.

Chapter three studies the property of genuine multiparty entanglement (that is, entanglement that cannot be understood as coming from entanglement of fewer systems), of stabiliser states and subspaces. The

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main boast of the associated article is the construction of genuine multiparty entangled subspaces (GME) that are NPT (non-positive partial transpose), which is indeed a novel and powerful result. This is achieved through several clever steps, notably including the introduction of an elegant representation of stabilisers in terms of vectors which captures the entanglement properties in a pleasing and complete way. This tool, which plays an important part of the proof, will likely be useful in other studies. In this work, it is used to prove necessary and sufficient conditions for stabiliser spaces to be GME in several forms, including non-commuting local stabilisers, and the NPT condition. These results are powerful and extensive, it's rather a tour de force of entanglement of stabiliser spaces, and, given the extensive role of stabiliser states and spaces in quantum information, it should help understand much more the role of entanglement, and GME in different applications. For example, I would be interested to understand what implications this had for any examples of stabiliser spaces that we use typically, such as stabiliser codes. A natural question would be do typical error correcting code spaces have this GME condition, or what kind of codes would?
The article itself is very nicely written, the results are concisely and clearly presented and it was published in Quantum, one of the top international journals of the field.

Chapter four extends the study to genuine multiparty non-locality, and shows that all GME stabiliser spaces are also genuinely multiparty non-local (GMNL) (again, the idea is that this means its non-locality cannot be understood as arising from non-locality of fewer parties). It does so by using results from the previous chapter, as well as existing work linking GMNL to the capacity to generate maximally entangled states across all bipartite cuts, for qubits, and new techniques for graph states in the qudit case. It requires refining the results from chapter three, as well as combining several ideas and techniques. The results indeed provide the first example of constructive GMNL spaces, as far as I am aware, which is very interesting. It has exciting perspectives as applications for device independence for example. It would be nice to see more discussion into the links to existing results, for example the known fact that for all qubit graph states it is known to be possible to locally achieve maximal entangled pair between any two parties, or that, again for qubits, any stabiliser state is locally equivalent to a graph state (presumably this is not true in the qudit case?). The article is nicely written, and appears on the public forum of the arxiv.

Chapter five makes the move to the network motivated setting of what is called Local Operations and Shared Randomness (LOSR), where parties are allowed to act locally and share classical randomness, but no communication is allowed. It is shown that in the LOSR setting, it is not possible to generate any non-trivial graph state (of three or more nodes) using only preshared bipartite entanglement. The usual setting studied in quantum networks allows local operations and classical communication (LOCC), where all classical communication is free. This chapter takes the other extreme and gives strong negative results, showing the power of classical communication in quantum networks. It provides several new and interesting techniques along the way. The new inflation technique used will likely be useful for different studies. It also identifies new graphical features of the graph states that facilitates its proofs (similarity of neighbourhoods of vertices), which also may be useful elsewhere. It is also shown that any state that is sufficiently close to a graph states cannot be created by LOSR. I would have been interested to know

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under what conditions, graph wise, the number of different inflations necessary becomes large. The article is very nicely written, and the introductory material in chapter two makes this chapter very accessible. It has been published in Nature's NPJ Quanutm Information.

Chapter six moves to non-locality in the LOSR setting, in particular the associated version of genuine multi-party non-locality (LOSR-GMNL), which is known to be different from the GMNL studied in chapter three. Several families of graph states are studied and proved LOSR-GMNL, depending on dimension - for qudits, all so called caterpillar states are LOSR-GMNL, for prime dimension all cluster states, and for arbitrary qudit, all GHZ states are LOSR-GMNL. This was proven using inflation techniques by introducing new inequalities. Finding new examples of GMNL is always interesting, in particular for such a motivated family of states, and it may be these features are related to their utility in quantum information. At some point in the comparison between the GMNL of Svectlichny (chapter 3), and LOSR-GMNL it is suggested the role of LOSR for the latter could be related to the role of LOCC for the former. This seems unclear to me (LOCC does not explicitly appear in the Svetlichny form, but LOSR is directly behind the definition of LOSR-GMNL). I would like to have seen more discussion on the links and rationale between LOCC and LOSR, similarly for the case of GME in both settings (could not an LOCC version of GME have overcome similar obstacles?). Then a new setting is presented, motivated by practical considerations, where some communication is allowed - Local Operations and Neighbour Communication (LONC), with a similarly defined version of genuine multiparty non-locality - LONC-GMNL. It is then shown that there are some states that are LOSR-GMNL but not LONC-GMNL. In a sense this is not surprising, since communication is allowed in the latter, but it is satisfying to see an explicit example. It would have been interesting to see how this work related to previous know works, particularly the notion of non-locality with communication introduced by Barrett et al (PRA 75(1):012103, 2007) which was behind the stated motivation for the definition of LONC, reference [38] in the article. The article is well written, and has been posted on the public forum quant-ph arxiv.

Chapter seven reviews the results of the previous chapters, and presents several interesting avenues of research and natural open questions, many beyond the questions I raised above, demonstrating a clear mastery of the domain and vision of the field.

It is my firm opinion that this manuscript is entirely of the level of a PhD and my recommendation that one be awarded on its basis.