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Summary of the doctoral thesis
"Genuine multipartite entanglement and nonlocality of quantum stabilizer states"

At its inception, quantum theory was not universally recognized as the correct description of physics at micro scales, since its many properties seemed to defy well-established principles of nature. Perhaps the most controversial in that regard was quantum entanglement as it seemingly requires an instantaneous influence between two spacelike separated events. This led to the development of an alternative model called the local hidden variable model, which aimed to explain the behavior of quantum states in terms of a theory obeying a set of principles called local realism. Ultimately, it was shown by Bell in 1964 that any theory obeying local realism is incompatible with quantum mechanics, i.e., that the behavior of some entangled states, called nonlocal, cannot be explained in terms of local hidden variables.

However, the same features that made entanglement and nonlocality so controversial also showcase their potential advantage in applications like computation or cryptography. For this reason, entanglement and nonlocality have been an object of intensive studies both for their practical relevance and as key features showing that quantum theory significantly departs from classical physics. While the bipartite case is already quite well characterized, the more general, multipartite scenario remains much less understood.

In this thesis, we study two notions of multipartite entanglement and nonlocality: the standard ones typically used in the literature and those originating from a scenario called local operations and shared randomness network (LOSR-network), which is based on a network composed of an arbitrary number of parties, sources of non-signaling correlations, in which the parties cannot communicate classically, yet they can share some classical correlations in the form of shared randomness. Notably, we concentrate on the most extreme, and at the same time the most valuable forms, of these correlations called genuine entanglement and genuine nonlocality. In our efforts, we narrow our attention down to quantum states originating from the qudit stabilizer formalism which is known for its use in constructing quantum error correction codes. This formalism provides an efficient and very convenient-to-handle description of multipartite quantum systems, and yet it is broad enough to include several important classes of states such as the graph states.

First, within the standard definition of entanglement, we introduce a general framework allowing us to detect and characterize entanglement in the qudit stabilizer formalism. Bulding on it we provide a necessary and sufficient criterion for a qudit stabilizer subspace to be genuinely multipartite entangled, i.e., that every pure state from the subspace as well as any mixed state defined on it, is genuinely multipartite entangled. We then prove that a stabilizer subspace is genuinely multipartite entangled if, and only if every partial transpose of any mixed quantum state acting on the subspace is non-positive. Interestingly, these results could suggest that there are no genuinely multipartite entangled states with positive partial transpositions in the stabilizer formalism.

Next, we show that each genuinely multipartite entangled, qubit stabilizer subspace is also genuinely multipartite nonlocal. In fact, we proved an even stronger result, showing that every pure state from, and every mixed state defined on, a genuinely multipartite entangled stabilizer subspace can produce measurement correlations in a Bell-type experiment that are nonlocal in the strongest possible sense, meaning that they have no contribution originating from any type of local hidden variable model.

In the second part of the thesis, we turn to the aforementioned LOSR-network model. Operating in this model, we have shown that no qudit graph state of prime local dimension can be generated by an LOSR-network with bipartite sources of quantum states. This, on the one hand, means that the entanglement depth of graph states in the LOSR-network based notion of entanglement is at least three. On the other hand, our result implies that classical communication is necessary to prepare the graph states in quantum networks powered with bipartite sources. Moreover, we have shown that some subclasses of qudit graph states, namely qubit caterpillar graph states, qudit cluster states of prime local dimension, and qudit Greenberger-Horne-Zeilinger states, give rise to genuinely multipartite nonlocal correlations in the LOSR-network model. This result establishes the equivalence of genuine entanglement and nonlocality in the LOSR-network model for these particular classes of states. In deriving the results concerning the quantum networks, the inflation method was heavily exploited.

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