This is a review of the dissertation **"Certification of quantum states and measurements in contextuality scenarios"** by **Rafael Freitas dos Santos**.

The thesis is based on the following scientific publications:

1. D. Saha, **R. Santos**, and R. Augusiak, Sum-of-squares decompositions for a family of noncontextuality inequalities and self-testing of quantum devices, *Quantum* **4**, 302 (2020).

2. **R. Santos**, J. Chellasamy, and R. Augusiak, Scalable noncontextuality inequalities and certification of multiqubit quantum systems, *Physical Review A* **106**, 012431 (2022).

3. **R. Santos**, D. Saha, F. Baccari, and R. Augusiak, Scalable Bell inequalities for graph states of arbitrary prime local dimension and self-testing, *New Journal of Physics* **25**, 063018 (2023).

The thesis has five chapters. **Chapter 1** is an introduction to probabilistic theories, non-contextual hidden-variable models, non-contextuality inequalities (CHSH and KCBS are used as initial examples), state-dependent and state-independent quantum contextuality, Bell nonlocality, graph states, and self-testing.

Chapter 2 is based on the scientific publication No. 1 above. It takes the family of tight noncontextuality inequalities associated to the *n*-cycle scenario and obtain their sum-of-squares decompositions. This allows to identify a set of algebraic relations that are necessarily satisfied by any state and measures that reach the maximum quantum violation. This is then used to prove that the maximum quantum violation of these inequalities can be

used for self-testing three-dimensional quantum states and measurements under weaker assumptions than those considered in Kochen-Specker contextuality.

Chapter 3 is based on the scientific publication No. 2. Here, a family of non-contextuality inequalities is presented that can be used for the certification of multi-qubit quantum systems without requiring spatial separation between subsystems (in return, it assumes certain compatibility relations between the measurements). The number of expectation values to be measured scales polynomially with the number of qubits that are certified. The robustness of the method versus experimental errors and imperfections is proven in one case (which makes sense, since these proofs are case dependent).

Chapter 4 is based on the scientific publication No. 3. Here, a family of Bell inequalities violated by graph states of any local prime dimension is introduced. The maximum quantum violations are analytically determined. An interesting feature of this family is that the number of expectation values required to check the violation only scales linearly with the size of the system. Another interesting feature is that some of these inequalities can be used for self-testing important quantum states such as the four-qutrit absolutely maximally entangled state.

Finally, **chapter 5** contains some brief concluding remarks, including directions for further research inspired by the findings presented in the previous chapters.

I have the following questions regarding the thesis:

Question 1. The sum-of-squares decomposition of the 5-cycle scenario in Reference 2 holds without the idealization that the measurements satisfy the commutativity conditions that characterize the 5-cycle scenario. To what extent would the same conclusion be valid for other scenarios?

Question 2. The self-testing statement in Reference 2 relies on the assumption that "the measurement device returns only the post-measurement system and has no memory". Does it rely on any assumption on the post-measurement *state* of the post-measurement system? Would it hold if the post-measurement state is maximally mixed?

Question 3. In Reference 3, it is said that "in Theorem 1 we do not assume the whole system to be composite". I interpret this as that measurements A_i and B_j with $i, j \in \{1, 2, 3\}$ in Theorem 1 do not have to be single-qubit measurements. Is this correct? If so, could you provide an example?

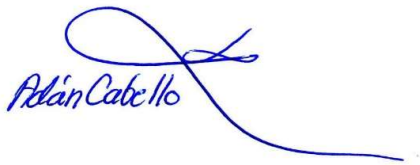
Question 4. N-qubit graph states are known to be self-testable. What is the difference between the number of contexts needed to self-test a given N-qubit graph state with existing methods and the number of contexts needed with the method in Reference 3?

Question 5. Can the findings in Reference 4 be used to self-test the three-qutrit "supersinglet" state also known as the Aharonov state [M. Fitzi *et al.,* Phys. Rev. Lett. **87**, 217901 (2001)]?

Question 6. Is there an analytical form of the classical bound of the Bell inequality in Eq. (49) in Reference 4 (written as 7.63816)?

The doctoral dissertation fully meets all the conditions specified in Article 187(1-2) of the Act. **Therefore, I conclude that the presented dissertation meets the formal requirements for a PhD thesis, and I recommend the admission of the Candidate to the subsequent stages of the procedure, including the public defense**.

In Sevilla, on December 15, 2023,



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Konkluzja recenzji rozprawy doktorskiej
(Conclusion of dissertation review)

Tytuł rozprawy *(Dissertation title)*: **„ Certification of quantum states and measurements in contextuality scenarios”**

Autor rozprawy *(Author of the dissertation)*: **Rafael Freitas dos Santos**

☒ Pozytywna ocena *(Positive conclusion)*:

Stwierdzam, że przedstawiona mi do recenzji rozprawa spełnia wszystkie wymagania ustawowe i zwyczajowe stawiane rozprawom doktorskim i wnoszę o dopuszczenie jej do dalszych etapów postępowania doktorskiego, uwzględniając publiczną obronę.

*(I conclude that the presented dissertation meets the formal and customary requirements for doctoral dissertations and I recommend its admission to subsequent stages of the procedure, including the public defense.)**

☐ Ocena negatywna *(negative conclusion)*

Stwierdzam, że przedstawiona mi do recenzji rozprawa nie spełnia wszystkich wymagań ustawowych i zwyczajowych stawianych rozprawom doktorskim i dlatego nie rekomenduję dopuszczenia jej do dalszych etapów postępowania doktorskiego.

*(I conclude that the presented dissertation does not meet the formal and customary requirements for doctoral dissertations and therefore I do not recommend its admission to subsequent stages of the doctoral procedure.)**

Uzasadnienie powyższej oceny znajduje się w raporcie będącym załącznikiem 1.
(The justification of the above assessment can be found in the detailed report in the attachment 1.)

09/12/2023
Data i podpis
(Date and signature)

Załącznik 1: Recenzja rozprawy doktorskiej
(Attachment 1: Review of the dissertation)

Zaznacz ocenę (Please tick the box with your conclusion)