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Review of the thesis entitled

Operational Quantum Frames

An operational approach to quantum reference frames

Submitted by

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The issue of the quantum reference frame, which has a long history, is now attracting the attention of the quantum foundation community. The author studies the topic with great rigor and generality.

Chapter 1 serves as an introduction, where the author explains the motivations and presents the underlying principles of the entire thesis. These principles consist of four items: operationality, relativity of measurement, gauge invariance and frame-covariance, and universality of quantum mechanics. Following the presentation, a detailed summary of the entire thesis is provided.

Chapter 2 is dedicated to introducing preliminary topics that are essential for understanding the rest of the thesis. An important aspect is the mathematical formulation of operational equivalence. The author introduces the operational state space as a quotient of the original total state space, divided by an equivalence relation: a pair of states is considered equivalent if they cannot be distinguished within a class of restricted effects. The operational state space is demonstrated to be a convex set in a Banach space, making it compatible with a general probability theory framework. In Section 2.2, the notion of covariant POVM is introduced, along with some accompanying observations. Section 2.3 focuses on the introduction of localizability as a property of the covariant POVM, which coincides with the so-called norm-1 property.

Chapter 3 provides a comprehensive description of the concept of relational quantum kinematics. To the best of my knowledge, this presentation is the first to offer both clarity and mathematical rigor, although previous incomplete versions have been mentioned in the literature. The author introduces several descriptions: invariant description and framed description, and combines them to define a relational description.

In Section 3.1, the author considers a composite system consisting of R and S, on each of which the group G acts. The set of all invariant effects is then identified with the set of measurable quantities. Moving on to Section 3.2, the system R is assumed to yield a covariant POVM called as a frame observable, which qualifies R as a quantum reference frame.

Section 3.3 introduces a novel notion called framed description. Here, the set of measurable quantities is restricted to elements that can be expressed as linear combinations of products between the frame observable and an effect of the system. Continuing to Section 3.4, the relational description is introduced by combining the invariant and framed descriptions. A map, denoted as Yen, plays a crucial role in producing relational quantities by relativizing an effect on S. Additionally, this map is used to introduce yet another description called relative description. The author further explores the relationship between the relational and relative descriptions.

In Chapter 4, the author explores the process of obtaining a system-only description. A crucial component in this endeavor is the restriction map, which plays a central role. The author demonstrates that if the reference frame is localizable, the standard system-only description can be obtained by considering localized reference states.

Section 5 focuses on the study of frame-change maps. This section presents entirely new results developed by the author. While previous research has primarily examined a single reference frame, this section expands the analysis to encompass two frames. The highlight of the thesis is Theorem 5.2.2 in Section 5.2. Here, the author introduces a frame-change map and demonstrates its consistency with a global state. In Section 5.3, the proposed map is compared to relevant works in the field.

Chapter 6 offers an extensive list of future perspectives.

There are several different approaches to addressing the issue, and one of them is the 'symmetry-based' approach, which the author adopts as the starting point in this thesis. The thesis provides a well-organized and rigorous description of this approach, which is considered the best among existing works. The author introduces the principles and examines their consequences without introducing any additional assumptions. These results are presented as mathematical theorems. Notably, the author demonstrates exceptional rigor in dealing with mathematically delicate points, a remarkable merit of this work. The author's mathematical skillfulness allows for the generalization of some known results. Moreover, in a noteworthy contribution from a physical standpoint, the author introduces the concept of the frame-change map for the first time.

I would like to highlight some areas for improvement or weaknesses in the thesis. Throughout the thesis, it would be beneficial to include examples. For instance, providing examples such as G=R (translation group) or G=R^2 (translation and boost group) for a single particle system could greatly assist readers. These examples can help to illustrate and reinforce the four principles presented in Chapter 1, making it easier for readers to understand the mathematical results in subsequent chapters. Additionally, from a mathematical perspective, considering concrete groups and their representations could lead to interesting problems, such as exploring projective representation versus true representation, or irreducible versus reducible representations.

The author's proposal to understand relative (relational) descriptions in terms of general probability theory is intriguing. However, it would be natural to investigate the geometry of the convex state space in this context. One significant question that remains is the limitation imposed by quantumness. In this thesis, the author primarily focuses on localizable reference frames. On the other hand, the consideration of finite quantum reference frames (or the semi-boundedness of generators) does not allow complete localizability. Therefore, it would be beneficial to explore the notion of "better" and "worse" reference frames. For example, generalizing the frame change maps to accommodate changes from "better" to "worse" frames would be valuable.

In summary, I conclude that the presented dissertation meets the formal requirements for a PhD thesis and recommend admission of the candidate to the subsequent stages of the procedure, including the public defense.