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Review of the PhD thesis of Jan Głowacki: Operational Quantum Frames

This is an overall high-quality PhD thesis. It develops a novel operational framework for quantum reference frames (QRFs) by building upon previous work by Busch, Loveridge and Miyadera and taking inspiration from a number of other recent developments in the field. The output is the mathematically most rigorous and, in some (not all) ways, most general framework for QRFs, as far as I can see. It thus certainly represents an original and valuable contribution to the literature, making decisive progress in a particular direction in the theory of QRFs. Up to minor nitpicks, the technical construction and argumentation appears to be correct and I would like to congratulate the candidate for his scientific achievements. There are, however, a few unwarranted conclusions about the placement of this novel framework within the field of QRFs that should be revised.

In this review, I will begin with the broad picture and the strong points, before focusing on weaknesses and finally providing a list with minor comments and suggestions.

Achievements and strengths

While dating back decades, the theory of QRFs has recently seen a surge in activity, triggered by the construction of several frameworks for implementing a form of QRF covariance and thereby extending the relativity principle properly into the quantum regime. QRFs are reference frames associated with quantum systems and thus, in contrast to the usual notion of external background frames, internal and part of the problem one wishes to describe. Naturally arising questions concern how one may "jump into the perspective" of a quantum system, describing others relative to it, and how one may change from one internal QRF perspective to another one. The latter changes of internal perspective should be given by QRF transformations, generalizing classical notions of frame changes. These are complicated by the fact that the frames are now subject to quantum fluctuations themselves and may be in "relative superposition".

The present thesis sits in this context and pursues a new approach to these questions with its main achievement being a novel type of QRF transformations. What sets the approach of this thesis apart is its focus on operator-algebraic techniques and especially its assertion that invariance under the (gauge) group associated with the QRFs should be implemented at the level of algebras of observables, rather than states. Taking invariant observables as the primary ingredient, representing everything an observer could in principle have access to, the relevant states, instead, are operationally defined as those that can be distinguished by the invariant observables and thus amount to equivalence classes on the original global space of density matrices. Crucially, this does not in general imply that these states are invariant themselves. With this definition of states, the approach of the thesis sidesteps the challenge arising for non-compact groups that invariant states cannot be part of the original space of states and rather have to be understood in a distributional sense.

This stands in contrast to the so-called perspective-neutral approach to QRF covariance. This approach is inspired by the quantization of gauge theories and thus, besides also focusing on invariant observables, further requires invariant states, in other words states that satisfy the gauge constraints. These constraints are crucial for the approach as the redundancy they give rise to is the whole reason why its QRF transformations turn out to be unitary. In the case of compact groups, the resulting states will be a subset of the states permitted by the approach of this thesis. For non-compact groups, the perspective-neutral approach invokes the construction of rigged Hilbert spaces and invariant states that are distributional, requiring thereby the construction of a novel inner product to be turned into a gauge-invariant Hilbert space; these are states not permitted in the present thesis. It is thus clear that the two approaches work with in general quite different state spaces and so the tools for QRF transformations of the perspective-neutral approach cannot be simply applied to the approach of this thesis, thus requiring a novel framework. Without imposing constraints, the question of invertibility of QRF transformations becomes a much more difficult one.

Similarly, the methods of the other earlier approach to QRF covariance, proposed by Giacomini et al and further developed by de la Hamette and Galley, cannot be applied to the framework of this thesis. This approach does not invoke invariance principles and is based directly on the states within internal QRF perspectives, essentially reading out the QRF transformation from the information about how one QRF describes another. Also the states considered in this approach are in general distinct from the ones admitted in the framework of this thesis.

The fact that the candidate, together with his collaborators, has established QRF transformations for this new approach is thus a highly non-trivial and original contribution. Furthermore, it is done at a level of mathematical rigor beyond any of the previous works on QRF covariance and for more general symmetry groups. The thesis also contains a comparison to the other approaches in the simple context of ideal QRFs where they are commonly defined, showing a non-trivial agreement (up to certain equivalences). In the general case, however, these approaches will not agree and as such the present thesis offers a genuinely distinct proposal.

Along the way, the thesis also develops a much more general notion of QRFs than in the previous literature (on QRF covariance). This refers not only to the class of symmetry groups considered, but also to the type of frame orientation observables considered. It is generally agreed on that the latter should transform "covariantly" under the group. The thesis invokes the most general class of such observables: covariant positive operator-valued measures (POVMs). While covariant POVMs have appeared before in the literature on QRF covariance, it was so far a restricted class, namely the one generated by coherent states. The reason such restrictions were made is that it is then much simpler to perform conditionings on frame orientations. The thesis thus encompasses all previously employed notions of frame orientation observables, but by going beyond it also had to invent a replacement for the previously invoked conditionings on frame orientations. This has been achieved by a certain partial tracing over a frame state.

Using the (in some sense) most general definition of QRF orientation observables, the thesis moreover refines previous definitions of different *types* of QRFs (ideal, complete, sharp, ...) and introduces the key notion of localizability of a QRF. A QRF is localizable when there exist pure states such that the probability for the frame to be in a certain orientation is arbitrarily close to one. I have not seen this notion before and it seems like an operationally valid characterization of "orientable" frames (though I have questions about this, see below). The QRF transformations of the thesis are so far restricted to localizable frames and it remains an open question how they could be generalized beyond this case. (The perspective-neutral approach goes further in this regard as it does not require its covariant POVMs to be localizable.)

The thesis concludes with a number of interesting perspectives on further developments, relating, among others, to "overlapping" or more general notions of jointly measurable frames and to how QRFs fit into a spacetime picture. It is clear that this program can (and should) be further developed.

The above achievements reflect without a doubt that the candidate has both the technical and conceptual capabilities worthy of the award of a PhD degree. The formalism represents a highly advanced direction within the foundations of quantum theory that also touches on spacetime physics. While the presented work has been done in collaboration with two co-authors, the thesis contains independent material not appearing in the published arXiv preprint, both in technical form, as well as in discussing the context and future directions. Furthermore, the material that does appear in the joint preprint [24] is presented in different and extended form in this thesis. The candidate has collaborated with different sets of researchers based at different institutions and has in particular not co-authored a paper with his official PhD supervisor so far. Altogether, this thesis and its context thus demonstrate the capability of the candidate to carry out scientific work independently, in fact rather suggesting a somewhat unusual level of independence for a PhD student. In conjunction, the achievements of the candidate therefore meet, in my opinion, the requirements for the award of a PhD degree.

Weak points

Let me now come to the weak points of the presented thesis. These concern unwarranted conclusions drawn by the candidate about how the presented approach relates to the other approaches to QRF covariance and what distinguishes it. The thesis is somewhat unjustifiably dismissive of other approaches and gives the impression that the candidate aims to establish it as the "right" approach. While understandable upon the establishment of the new framework, QRF covariance is not a question of "right" or "wrong" religion. Although all address the question of QRF covariance, the different approaches do so in distinct contexts and come with different physical premises and capabilities and thus with distinct regimes of validity. Specifically, there is not one single "right" approach to QRFs. For example, the older quantum information approach does not lend itself easily to QRF covariance, in contrast to the newer approaches especially designed for that. Conversely, for operational tasks of dealing with scenarios where different agents do not share a common external reference frame and wish to communicate quantum systems nevertheless, it is the quantum information approach that is the suitable one, while the earlier approaches developed for QRF covariance would lead to unnatural state restrictions in that case (this comparison is further discussed in Ref [36]).

In particular, the perspective-neutral approach that the candidate criticizes in several places throughout the thesis is a perfectly valid approach to QRF covariance. It applies to contexts where it is correct to implement gauge constraints, hence in particular in gauge theories and gravity, but also in certain situations in quantum foundations. For example, the perspective-neutral approach is capable to treat internal QRFs in QED where the frames will then be associated with the U(1) gauge group (e.g. they can be associated with edge modes or the longitudinal part of the connection) and it is physically *necessary* to implement the constraints. This is after all why the photon has two propagating degrees of freedom. If one did not implement the constraints, one would simply not obtain the right number of physical degrees of freedom and states that don't are not physical. The same is true for the other interactions of the Standard Model and it is also true in gravity. In this context, the approach of the thesis would in fact not be the appropriate one. There are other contexts in quantum foundations where this would also be true. Of course, this does not at all invalidate the approach of this thesis, which describes different contexts, though the onus is on the candidate and his collaborators to characterize the physical situations when this operational approach is the appropriate one to invoke.

Coming back to gauge theories, the candidate stresses on page 4 that the *"[...] implementation [of gauge invariance in the new approach] is very different [from the perspective-neutral one] – in the presented framework, in analogy to gauge theories, it concerns observables rather than vectors in $\mathcal{H}$."* This statement is not correct: in gauge theories one in fact *also* imposes gauge invariance of the states, i.e. one implements the constraints also at the state level. That is exactly what the perspective-neutral approach does. As alluded to above, this is necessary for obtaining the correct number of degrees of freedom and true both classically and quantumly. Indeed, imposing gauge invariance in those theories is more than just considering invariant observables and it is also not only a kinematical action. Classically, this can be seen from the fact that the space of solutions of

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the theory automatically solves the constraints and at the end of the day the only observables that matter are the gauge-invariant ones on that space. It is thus the contrary: it is the perspective-neutral approach that is close to gauge theories. This is by construction as it uses the corresponding quantization techniques.

Next, the author claims to formulate four physical principles on pages 3 and 4 that would provide a clear conceptual basis for the approach of the thesis. The candidate also plans to show that these principles single out the presented approach. With the present form of the principles, this cannot work, as all of them can also be claimed to be satisfied by the perspective-neutral approach. In other words, the approach of the thesis is just one possible interpretation of how to mathematically implement those principles. For example, as regards **operationality**, also the perspective-neutral approach only concerns observable quantities and one can phrase that in terms of gauge-invariant (conditional) probabilities. It is true that the set of observables is not the same as in the thesis, but the principle as stated does not specify which set it should be.[1] In fact, whatever is operationally testable and falsifiable depends on the scenario at hand. For example, in actual gauge theories, it is the set of observables consistent with the perspective-neutral approach. As regards **relativity of measurement**, this is also true in the perspective-neutral approach: all physical observables are relational. Similarly, **gauge-invariance and frame-covariance** hold by construction in the perspective-neutral approach. Lastly, **universality of quantum mechanics** is also clearly obeyed in the perspective-neutral approach in the way this principle is phrased: its states form the complete set of density matrices over a Hilbert space, the physical Hilbert space. (The same is not true in this thesis where the admissible states form only a subset of states over a Hilbert space.) The principles in their present form can thus not be used to claim that the approach of the thesis is conceptually better motivated than others.

The candidate also views it as a shortcoming of other approaches that they have not been developed for arbitrary topological groups, in contrast to the work done in this thesis. It is true that this has not been done for other approaches, but that can hardly be viewed as a shortcoming per se of these approaches. The fact that it has not been done yet does not mean that it cannot be done. The candidate writes on page 20 *"Another advantage of the presented approach to invariance, as compared to PN, is its greater generality as we do not need to assume unimodularity of G."* This is incorrect: for practical purposes unimodularity has been assumed in [8] as a sufficient condition, but it is *not necessary*. Non-unimodular groups can also be encompassed by the perspective-neutral approach upon a slight generalization of the notion of invariant states (this case has been discussed in the context of group averaging). There is a similarly pessimistic statement on page 3 and both should be revised. In any case, what is mathematically possible is not always necessarily physically relevant. Which non-unimodular group is of direct physical relevance? There are infinite dimensional gauge groups in field theories, such as the diffeomorphism group, but these can currently neither be incorporated in the work of this thesis.

Let me now come to the notion of localizability of a frame, which indeed is a very nice and operationally useful notion. The thesis provides a nice characterization of such frames in terms of the existence of a sequence of pure states that converges to a Dirac delta measure. But then the candidate claims on pages 7 and 22 that in the context of coherent state based covariant POVMs, as used in the perspective-neutral approach, the frame can only be localizable if its coherent states are sharp, i.e. perfectly distinguishable. Is there a theorem that states this? It does not seem to obviously follow from what is presented in the thesis. If this statement is correct, then clearly this notion of localizability is physically not useful, despite being mathematically nice. For example, it would then call Gaussian coherent states non-localizable, which clearly provide a simple and physically very useful notion of peakedness and localization. This would then also render the restriction to localizable states for QRF changes of this approach an ultra strong one. But from the

[1] In a sense, the observables of the perspective-neutral approach are less, as only the support of invariant-observables on the physical Hilbert space is considered. In another sense, however, this approach also admits more invariant observables. This is because observables only have to commute with the constraints *up to terms proportional to the constraints*. Outside the physical Hilbert space, this would not necessarily give rise to invariant operators, on the physical Hilbert space it does, however.

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Gaussian states also the question arises: can one not build sequences of Gaussian pure states with smaller and smaller variance that ultimately converge to a Dirac delta, thus actually fitting the bill for localizability? If so, then the statement by the author is wrong that localizability for coherent state based QRF models is only possible when they are sharp. Hence, it appears that either the statement is incorrect or the invoked notion of localizability is physically too strong. In any case, it is interesting to note that the perspective-neutral approach does not require localizable states for QRF transformations to be unitary.

In this context, the author also claims that it is only possible for countable groups that one has perfectly distinguishable coherent states. If these states are supposed to be elements of the QRF Hilbert space, then this is true. It is, however, not true that this is the requirement for the notion of coherent states in the covariant POVMs of the perspective-neutral approach. Indeed, the last paragraph on page 12 in [8] expressly states that distributional notions of coherent states are permitted (see also Examples 2 and 5).[2] For example, in this way position eigenstates are permitted for the translation group and this can certainly be treated rigorously and these states clearly are perfectly distinguishable. The statement by the candidate that coherent system POVMs can be sharp only in the *"very simple setup of a countable group"* are thus inaccurate and a misrepresentation of what is done in [8]. Based on this, the candidate writes *"We, therefore, stick to our definition of ideal frames which has a much greater domain of applicability."* In view of what I just wrote, this is clearly too strong a statement, despite the fact that more general groups are permitted. Further based on this, the candidate also writes on page 22 *"We avoid the use of coherent state systems in our formalism [...]"* (and something similar in footnote 3 on page 15) as a contradistinction against the perspective-neutral approach. In fact, this is wrong: the formalism does not avoid coherent states, it *encompasses* them; coherent system POVMs constitute an example of the general covariant POVMs admitted by the very general approach for QRF observables presented here. This whole argumentation is thus rather odd and should be revised. Related statements are on page 47 and also there the statement that the coherent states need to be Hilbert space elements in [8] should be corrected.

Similarly, on page 22 the candidate writes *"Indeed, [coherent state systems] constitute a* defini*tion of a frame in [8]"*. This is a misrepresentation of the perspective-neutral approach and should be corrected. It is not true that the definition of QRF in that approach is restricted to coherent states systems and that is also not stated anywhere in [8]. Instead, in [8] the consideration is, for practical purposes, restricted to *modeling* a QRF via a coherent state POVM, but there is no a priori reason why one should not be able to consider more general covariant POVMs along the lines of what is done in this thesis and this would be an interesting endeavor. Somewhat related: footnote 3 on page 15 seems to suggest that the coherent system POVMs in [8] were based on irreducible representations. But that is expressly not the case and a slight rewording would be appropriate.

Since the candidate highlights inspiration by special relativity for the approach to QRF covariance of this thesis on page 38 and 48, while insisting on it being conceptually better motivated than previous approaches, let me stress that the perspective-neutral approach in fact admits a more explicit relation to the structure of special covariance that is explained in some detail [8] (see e.g. Sec. 2). This is just to highlight that it would be appropriate to be more careful with some of the statements in this thesis.

On page 47, the author finally criticizes *"the strong claims about the 'relativity of superpositions and entanglement' that can be found e.g. in [10] [...]"*. By contrast, I would probably phrase the discussion on page 47 more carefully. Within the approach presented in the thesis, it is indeed true that states showcasing the 'relativity of superpositions and entanglement' are operationally indistinguishable from states do not showcase this. The reason is the operational restriction to using only a specific POVM as observables for one of the frames. This gives rise to a superselection: with only a small subalgebra of observables at hand, one cannot distinguish certain superpositions from mixtures. But this being true in this thesis cannot be used as an argument against the

[2] It is true though that the imprecise discussion on page 10 of [8] could be read as implying that coherent states should be Hilbert space elements, which however is not the case.

statements in [10]. Why should the second frame observables only be restricted to one POVM? This does not appear to be a physical necessity and in cases where this restriction does not apply the claims in [10] are certainly valid.

Let me stress that *none* of the above weak points of this thesis actually appear in the corresponding arXiv preprint [24], which is much more carefully worded and does not feature unwarranted claims. They have thus been inserted by the candidate during the writing of this thesis and I would suggest a revision of these statements.

Comments and nitpicks

Finally, I list a couple of comments in bullet points and minor nitpicks for improving the presentation:

- Proposition 4.4.3 on page 35: is the word $*$-isomorphism correct in the first part of the statement? Usually, in the context of C^*-algebras this term is reserved for an invertible $*$-homomorphism, but in the first part of the statement the relation does not seem to be a homomorphism. Indeed, the second statement is that if the frame is further sharp, the map is multiplicative (hence a homomorphism). Should it instead perhaps be something like an *isometric $*$-bijection* in the first part?

  A related comment on this: for a relativization map, it would in fact be quite natural to *require* it to be a unital $*$-homomorphism, something that here is only true for sharp frames. It would be natural as this means all the algebraic structures are preserved in the relative description. In the perspective-neutral approach the relativization map constitutes such a unital $*$-homomorphism when the input system observables are restricted to be compatible with solutions to the constraints, regardless of whether the frame is localizable or not (see Theorem 1 in [8]). Is something similar possible here?

- The symbol $\mathcal{R}$ on the top of page 5 is undefined

- Page 5, 2nd paragraph: "$X \mapsto g.X$ for $h \in G$" $\longrightarrow$ "$X \mapsto g.X$ for $g \in G$"

- Page 9, 30 and 38: it is claimed that the lifting map is an analog of the disentangling map in [8]. I don't think this is true. The domain of the lifting map $\mathcal{S}(\mathcal{H}_\mathcal{S})^\mathcal{R}$ rather seems like the analog of the physical system Hilbert space $\mathcal{H}_S^{\text{phys}}$ of the perspective-neutral approach. As it maps into the space of states that are distinguishable by invariant observables, the analog of the physical Hilbert space $\mathcal{H}_{\text{phys}}$, it seems to me rather that the lifting map is an analog of the inverse reduction maps $\mathcal{R}^{-1}$ in the perspective-neutral approach in either the Schrödinger or Heisenberg picture. By contrast, the disentangler takes physical states in $\mathcal{H}_{\text{phys}}$ and maps them into certain gauge-fixed states where the frame part is factored out.

- Page 11, 2nd paragraph: *universality of quantum mechanics principle (III)* should be (IV).

- Page 13: the definition of operational equivalence seems related to the definition of "observational equivalence" in [36], Def 13, at least when restricted to invariant observables.

- Page 19, last paragraph and page 20, 2nd to last paragraph: the candidate writes that for G non-compact the space of invariant states would be empty and that this would justify the approach of this work. However, I don't think this is entirely correct. Indeed, invariant states will exist as distributions in those cases and one could work with a direct integral rather than direct sum decomposition. With this insistence, one may simply miss some interesting notions of states. I understand that the author would like the states to be contained within the original set of states he started out with and this could be phrased more clearly. But it is closing the door to potentially interesting and physically relevant notions of states.

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- Page 26, footnote 3: I am note sure I understand what is intended with the comment about the twirl map.
- Page 28: the candidate introduces the terminology of "relational" and "relative" descriptions, something not done in the corresponding paper [24]. I would also suggest a revision of this terminology for the simple reason that "relational" and "relative" mean the same thing to most people and this can thus only lead to confusion.
- Page 29 and 49: the candidate also introduces the notion of "perspective-independent description", something not done in [24]. Also this seems unfortunate terminology, given that the word "perspective-neutral", which again means essentially the same, is already used for a very similar structure in the literature.
- Page 32: why are the product-relative states a generalization of alignable states? The former are states on only the system, the latter are global.
- Page 39: it is stated that one can make sense of the states $|0\rangle$ of the approach in [10] via localizing sequences of states, but that it would not be clear that this could be done for the relevant structures of the perspective-neutral approach. Why can one not use these sequences also to make the notion of conditioning physical states precise, i.e. one uses localizing sequences of conditionings? This would then similarly make precise the reduction maps of the perspective-neutral approach.
- Page 42, line 12: in the proof a footnote has become part of the main text.
- Page 51: the candidate writes *"we believe that the (sufficiently developed) operational frame-change maps will provide a final resolution of the conceptual difficulties connected to the Wigner's Friend type scenarios [...]"*. This may be overly optimistic. One needs to include measurements in this approach and, at least for invertible QRF changes, a projective measurement in one perspective will map into a projective measurement in the other. It is thus unclear how a Wigner's Friend scenario could be encompassed, where one perspective sees a measurement, while the other only sees unitary evolution.
- References [7] and [36] are the same, the former is the arXiv version, the latter the published one.

Verdict

The list of weak points and nitpicks shall not cloud the fact that altogether this is a high-quality work that constitutes a significant contribution to the theory of QRFs and I congratulate the candidate for his achievements. I suggest the candidate, however, to revise some of the above mentioned unwarranted claims into more careful statements in order to improve the overall presentation of the work.

Therefore I conclude that the presented dissertation meets the formal requirements for a PhD thesis and recommend admission of the candidate to the subsequent stages of the procedure, including the public defence.

Yours sincerely,

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