Abstract

This thesis provides a conceptual basis and a formal machinery for a *relational* and *operational* approach to the foundations of quantum theory. The framework builds on the ideas of so-called *quantum reference frames* (QRF) and previous work on *quantum measurement theory* in the presence of symmetries. In a nutshell, the QRF program is based on the idea that reference frames should be treated as physical systems, combined with an assumption that as such they should be modeled within quantum mechanics. This perspective is aligned with insisting on relationality in physics, which is understood as justifying the fact that observations are always made with respect to some other, reference, system. Broadly speaking, physics should then be primarily concerned with relations between physical systems. In this work, we combine these insights with the emphasis on operationality, understood as refraining from introducing into the framework objects not directly related to in principle verifiable probabilities of measurement outcomes, and identifying the setups indistinguishable as such. Combining these insights with intuitions from special relativity and gauge theory, we introduce an operational notion of a quantum reference frame—which is defined as a *quantum system equipped with a covariant positive operator-valued measure* (POVM)—and build a framework based on the concept of *operational equivalence* that allows us to enforce operationality by quotienting the quantum state spaces with equivalence relation of indistinguishability by the available effects, assumed to be *invariant under gauge transformations*, and *framed* in the sense of respecting the choice of the frame's POVM. Such effects are accessed via the *yen* ($\yen^{\mathcal{R}}$) construction introduced in previous work, which maps effects on the system to those on the composite system, satisfying gauge invariance and framing. Such effects are called *relative*, and the classes of states indistinguishable by them are referred to as relative states. These can be identified with states on the system since they correspond to the image of the predual map $\yen^{\mathcal{R}}_*$. We show that when the frame is *localizable*, meaning that it allows for states that give rise to a highly localized probability distribution of the frame's observable, by restricting the relative description upon such localized frame preparation we recover the usual, non-relational formalism of quantum mechanics. We also provide a consistent way of translating between different relative descriptions by means of frame-change maps, and compare these with the corresponding notions in other approaches to QRFs, establishing *operational agreement* in the domain of common applicability.