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Review of the doctoral dissertation by MSc Julius Serbenta "Bi-local geodesic operators as a tool of investigating the optical properties of spacetimes"

The doctoral dissertation of Mr Serbenta has been written under the supervision of Prof. Mikołaj Korzyński. The thesis concerns an alternative formalism for determining the optical observables in curved spacetimes. This formalism, named the bilocal geodesic operator (BGO) formalism, has been developed by Prof. Korzyński, his group (Mr Serbenta, Mr Grasso) and collaborators. The doctoral dissertation focuses on applications of the formalism. The main results of the thesis have been already published in two papers co-authored by Mr Serbenta (these articles are included in the body of the thesis).

The problem studied in the thesis is directly related to the interpretation of astronomical observations within general relativity. (In fact, the studied formalism has much wider range of applications — it applies to any metric gravity theory.) Majority of astronomical observations are interpreted in the geometric optics approximation with electromagnetic waves treated as a collection of light rays forming a congruence. Over the course of last one hundred years, several different formalisms have been developed to study geometrical optics in curved spacetimes. The advantage of the BGO formalism comes from the fact that it extends the standard set observables by including the effects related to variations of the observer's position and/or an observation time (e.g. the parallax and the drift). In realistic settings, these effects are rather subtle. Nevertheless, they are a

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valuable source of information in the coming era of the precision cosmology. These effect are hard to study wth standard, unadapted to the problem methods. Therefore, the BGO formalism and the questions raised in the thesis could be very relevant for interpretation of the astronomical data in a not so distant future. The contribution of Mr Serbenta, presented in the thesis, is related to applications of the BGO formalism. In order to evaluate the contribution of the PhD candidate in more detail, I describe below the structure of the dissertation and the main results.

The thesis consist of five chapters and has a form of the stapler thesis. It starts with a brief introductory chapter followed by the chapter with mathematical preliminaries which are necessary to understand the BGO formalism (this chapter contains also some nontrivial original calculations). The main results are presented in chapters three and four which include the articles co-authored by Mr Serbenta. These chapters contain also short intruductions and descriptions of authors' contributions. Their pervading theme are applications of the BGO formalism. In the final fifth chapter (followed by the bibliography and two appendices with technical details), the candidate summarizes the results and outlines the directions for future research.

The introduction is well written and the motivation to conduct the studies is explained clearly. I find the exposition of the BGO formalism presented in the next chapter "Mathematical preliminaries" slightly "uneven." Mr Serbenta starts it with a rough explanation of very basic notions (such as a coordinate chart and tangent spaces), but then silently assumes that the reader is familiar with much more sophisticated mathematical machinery such as the exterior derivative, the Lie derivative, the interior and exterior products. This was probably an uneasy compromise between brevity and readibility of the text and does not alter my positive opinion about the style of the presentation. The second chapter contains also original results of the candidate who shows how to define the BGOs with the help of the geometry of the tangled bundle. The chapter three is based on the article co-authored by Mr Serbenta (the first author) and the supervisor. The article, included in the body of this chapter, has the self-explanatory title "Bilocal geodesic operators in static spherically-symmetric spacetimes." The authors derived the BGOs for a general highly-symmetrical spacetime using two methods out of which one was invented and implemented by the

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candidate. Next, they applied the formalism to the Schwarzschild spacetime and studied observables numerically. These results provide an important consistency check for the BGOs formalism. In the chapter four, a method to test the null energy conditions with precise distance measurements was presented. The article on this topic, co-authored by the supervisor and Mr Serbenta, is included in the body of the chapter. The main result of this paper, following from the application of the BGO formalism, is an inequality between the trigonometric parallax and the angular diameter distance which states that for a given source and an observer the trigonometric parallax distance is never shorter than the angular diameter distance provided that the null energy conditions holds and that there are no focal points in between. The inequality provides an interesting example of the application BGO formalism. The candidate's contribution to this research was important. The fifth chapter contains a summary of the presented results and expose possible directions of further studies.

I have no doubts that the doctoral dissertation of Mr Serbenta makes advances in the field of the geometrical optics in curved spacetimes. It is a well written coherent piece of work with the existing literature cited adequately. The results presented in the thesis were published in reputable journals: Classical and Quantum Gravity, Physical Review D. Moreover, Mr Serbenta co-authored two other papers published in Physical Review D.

I do not find serious omissions in the thesis, but I think it would benefit from being supplemented by a more detailed discussion of the observational status of the proposed tools: what is and what is not beyond the scope of the present day observations? This topic is partially addressed in the included papers, but an additional chapter devoted to it would clarify the situation. The candidate developed a tool, but an ultimate goal should be to apply it to real astronomical data. A time delayed flickering of multiple images of gravitationally lensed quasars is routinely observed by astronomers for many years now [Treu, Suyu, Marshall, Astron. Astrophys. Rev. 30, 8 (2022)]. Already in 1964, Refsdal suggested that this kind of effects may be used to infer cosmological parameters (MNRAS 128(4), 307-310, 1964). Recently, the rapidly growing catalog of known sources made a time-delay cosmography a valuable and precise cosmological tool that may shed some light on such fundamental problems as the

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so-called "Hubble tension." Is the BGO formalism applicable to these problems? Can it be useful?

The minor misprints and typographic errors spotted in the text: *esting* (page vi), *bi-local* — the inconsistent form with the remaining part of the thesis (in the title), too large white spaces after abbreviations [many instances, e.g. below Eq. (2.8)], typographic widows (pages 5 ,17), in the bibliography some journal names are abbreviated, but others are not (pages 72,76), many instances of the incorrect capitalization in the bibliography, incomplete bibliographic data of the published articles (the references [23], [118], [119], [120]), repetitions (the reference [52]). These errors do not alter my positive opinion about the quality of the presentation.

In my opinion, Mr Serbenta's doctoral dissertation contains novel results which are technically sound and important for our understanding of observations in curved spacetimes. The presented thesis demonstrates the candidate's ability to conduct independent research and his good understanding of the existing knowledge in the field. **The thesis submitted by Mr Serbenta meets the formal requirements applicable to doctoral dissertations in theoretical physics. It allows me to inference for admitting Julius Serbenta to further stages of the PhD process, including the public defense of his doctoral dissertation.**

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