REFEREE'S REPORT

On the PhD Thesis of Mr. Julius Serbenta

"Bilocal geodesic operators as a tool of investigating

the optical properties of spacetimes"

This Thesis resulted from a collaboration of Mr. Serbenta with Professor Mikołaj Korzyński on a novel approach to the description of propagation of light in curved spacetimes. The novelty consists in relating the geodesic deviation of light rays to potentially observable effects such as direction drift and redshift drift. This is a truly pioneering activity based on elegant mathematics and holds promise of new observational tests of cosmological models. It will probably become a broad field of research in general relativity.

The Thesis consists of two papers published jointly with Prof. Korzyński, in *Classical and Quantum Gravity* and in *Physical Review D*, extended for a separately written introduction and summary.

In the introduction, the author gives a very readable tutorial on tangent bundles and then presents the formalism of bilocal geodesic operators (BGOs). The application of the latter to geometric optics is largely an invention of Prof. Korzyński, developed in a series of papers written with collaborators (Mr. Serbenta among them). The BGOs, in brief, allow for calculating the geodesic deviation along a light ray out of initial data given at the position of the observer. This, in turn, allows for understanding the relation between observed quantities and the curvature of spacetime. The BGO formalism is exact (i.e., not based on linearised approximations) and independent of the theory of gravitation – no field equations of gravitation are assumed, just the metric.

In the first published paper, the authors apply these mathematical tools to calculate various observable quantities for light rays propagating near a black hole in the Schwarzschild spacetime. In an accessible, even attractive way, they describe the interplay between the parallax distance and angular diameter distance to the light source for an observer receiving rays bypassing a black hole with various impact parameters. This whole consideration has the potential to become useful in astronomical observations of black holes. This is, however, a matter of the future, perhaps distant future, because at the present level of observation technology it is a success to just prove that a black hole really is there at its hypothetical location.

In the second published paper the authors prove two theorems that, in abbreviation, say that the parallax distance can never be larger than the angular diameter distance if the null energy condition is fulfilled. It is remarkable that such a general result could be proved without any simplifying assumptions about the metric. This result, too, can be useful for future cosmological observations. The final section of the paper is devoted to the discussion of perspectives and problems connected with such an application. At present, problems dominate, but this may change in the future.

The authors carried out an impressively broad survey of literature related to their topic. The list of references includes even a few papers from the field of seismology, where methods similar to geometric optics are applied.

The Thesis contains a little gem from classic differential geometry: the simple but little known observation that every Killing vector field obeys the geodesic deviation equation. I found this observation so instructive and enlightening that I decided to include it in the new edition of the relativity textbook written by Jerzy Plebański and myself (of course, with a citation of Serbenta and Korzyński).

I noticed only two little imperfections in this Thesis that have no consequences for its conclusions. The first is the comment to eq. (23) in the CQG paper. In a spherically symmetric metric the choice of the component $C(r) = r^2$ does involve some loss of generality. This condition cannot be imposed when C is constant. Solutions with constant C are known for the Einstein and Einstein-Maxwell equations, these are the Nariai and the Bertotti-Robinson solutions, respectively, see Ref. [1] below.

The second little imperfection concerns terminology. What the authors call a surface-forming ray bundle (below eq. (83) in the CQG paper) is in fact a hypersurface-orthogonal bundle. Two vector fields u and v are called surface-forming when their integral lines form a congruence of surfaces; then the fields obey the equation $[u, v] = Au + Bv$ with scalar coefficients A and B.

In conclusion, I certify that the PhD Thesis of Mr. Julius Serbenta fulfils, with excess, all the legal and traditional requirements of PhD Theses and I postulate to admit it to further proceedings. I also propose to award a distinction to it – it boldly explores a novel field of research at a high level of expertise, using refined mathematical methods. This is an ideal combination for a work in theoretical physics.

REFERENCE

[1] A. Krasiński, *Gen. Relativ. Gravit.* **31**, 945 (1999).

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