Report on the PhD thesis of Julius Serbenta

Dr. Abraham I. Harte
School of Mathematical Sciences
Dublin City University

February 15, 2023

Overview

This thesis, "Bi-local geodesic operators as a tool of investigating the optical properties of spacetimes," describes a number of new results related to gravitational lensing in generic spacetimes. The main results are:

1. The computation of bi-local geodesic operators (BGOs) in static, spherically-symmetric spacetimes.

2. Numerical determination of parallax and angular diameter distances in Schwarzschild, as well as an analytic discussion of their behaviors in different regimes.

3. An inequality between parallax and angular diameter distances which holds assuming the null energy condition.

The thesis is composed mainly of two published papers which were jointly written with the candidate's supervisor, Prof. Mikołaj Korzyński. In addition, there is an explanation of what the candidate contributed towards those papers, some introductory material, and two brief appendices. The published papers involve a number of interesting results and are clearly written. Furthermore, the candidate's descriptions of his own contributions to these papers seem to be significant.

Weaknesses

The parts of the thesis which were written solely by the candidate are mainly a mathematical introduction to the BGOs, which are the central technical

1

tool in the thesis. These parts were not written as well as the published papers. They also seemed to contain very little physics, except for some promises that the concepts being described would be useful (or even crucial) for understanding the BGOs. Contrary to these claims, there didn't seem to be very substantial links made to the later material; the physics seems to be understandable without all of the extra mathematical terminology. The motivation for the mathematical section was therefore unclear.

Another weakness of the thesis is in its literature review. For example, Ref. [18] doesn't really use propagators in the same way as here, despite what is claimed. Propagators very similar to those used here have however been employed to understand lensing in plane wave spacetimes [Harte, "Strong lensing, plane gravitational waves and transient flashes" (2013) and Harte, "Optics in a nonlinear gravitational plane wave" (2015)], although these references aren't mentioned. It also seems to be implied that Jacobi propagators have mainly been used before only along timelike curves, which is not true.

Along the same lines, it is mentioned that Synge's world function can be used to understand optics, but except for [61], the given references are only general descriptions of Synge's function, unrelated to optics. Synge's function has nevertheless been employed in other optical problems, for example in the aforementioned plane wave references. Dixon's use of Jacobi propagators [19] is strongly tied to Synge's function as well.

The first paragraph of the conclusion claims that "the transfer matrix theory for the propagation of light as presented in the literature seems incomplete, as it cannot accommodate either redshift or position drift effects. The formalism presented here allows to consider all possible lowest-order nontrivial optical effects." This seems to be a crucial statement, yet as far as I can tell, it's not justified very well in the body of the thesis. There is no clear comparison with previous "transfer matrix" concepts, nor is there a clear and concrete discussion for why the BGOs are better. For that matter, there is no serious discussion of drift effects in the thesis. While there are some vague statements in these directions (sometimes with references), they are important enough that they should be thoroughly explained in a thesis such as this.

Summary

This thesis presents a powerful formalism to organize and understand different results in gravitational lensing, and uses it to derive several interesting

consequences. The work is interesting on its own and is also likely to inspire a significant amount of future work. The quality of the research presented here is therefore high, and certainly worthy of a Ph.D. While there are some weaknesses in the thesis, they are minor and mainly of an organizational character. *Therefore, I conclude that the presented dissertation meets the formal requirements for a Ph.D. thesis and recommend admission of the candidate to the subsequent stages of the procedure, including the public defense.*

Dr. Abraham I. Harte

Assistant Professor
School of Mathematical Sciences
Centre for Astrophysics and Relativity
Dublin City University

abraham.harte@dcu.ie