Report on the thesis

Certification of entangled quantum states and quantum measurements in Hilbert spaces of arbitrary dimension

by Shubhayan Sarkar

The thesis "Certification of entangled quantum states and quantum measurements in Hilbert spaces of arbitrary dimension" of Shubhayan Sarkar deals with the problem of certifying in a device-independent (DI) way, i.e. through Bell nonlocality, or one-sided-device-independent (1SDI) way, i.e. through quantum steering, entangled states and quantum measurements. This is a problem that in the last decade has become of central theoretical interest in quantum information science and that finds application in many DI or 1SDI protocols, such as randomness generation, quantum key distribution, and certified quantum computation. Many of the existing results in this area had previously been obtained in the case where Alice and Bob perform one of two dichotomic measurements in which case one can use Jordan's lemma to reduce the analysis to qubits, i.e., Hilbert spaces of dimension two. This thesis contains very general results that apply to systems of arbitrary dimension where the known qubit techniques cannot be used.

There are basically three main classes of results in the thesis, presented respectively in Chapters 3, 4, and 5 and which have given rise to four publications.

In chapter 3, a scheme is presented to self-test the *N*-partite GHZ state of local dimension *d*, for any $N, d \geq 2$ using Bell inequalities composed of two *d*-outcomes measurements per party. Specifically, it is demonstrated that the maximal violation of the ASTA Bell inequalities can be used to self-test the generalized GHZ state and the generalized CGLMP measurements. As a byproduct, it is shown that this scheme can be used to securely generate $\log_2 d$ bits of randomness, the maximum amount of randomness using projective measurements with *d* outcomes. This result is interesting from a foundational point of view as this is the first instance, where a single measurement can be used to generate genuine unbounded randomness (since *d* is arbitrary).

In chapter 4, a family of steering inequalities is introduced that are maximally violated by the maximally entangled state and any set of *d*-outcome genuinely incompatible projective measurements, such as mutually unbiased based. It is further demonstrated that these steering inequalities can certify this state and measurements and a robustness analysis is provided (in the specific case where the state is assumed to be pure and Bob's measurements projective).

Chapter 5 presents the construction of a family of steering inequalities maximally violated by any pure entangled state of local dimension d and two *d*–outcome measurements on each side. Using the maximal violation of the steering inequalities, it is then shown how to certify any pure bipartite entangled state and a pair of mutually unbiased bases of arbitrary dimension on the untrusted side. This is the first instance, where any pure bipartite entangled state can be certified using two measurements on each side, the minimal number of measurements required to observe quantum nonlocality. Additionally, it is demonstrated that this new protocol can certify any rank-one extremal measurement and that this task can be achieved with quantum states that are close to separable states, i.e., with a low level of entanglement. Finally, these results are used to devise a simple scheme for the certification of $2\log_2 d$ bits of randomness, the optimal amount using quantum systems of local dimension *d* in the 1SDI scenario.

All these results are impressive, both regarding their significance and impact on the field of self-testing and quantum certification and regarding the methods and techniques used to establish them. Furthermore, they open many interesting avenues for future work, such as those exposed in Section 6.2. Finally, I found the thesis very well-written and enjoyable to read.

In summary, I find Shubhayan Sarkar's PhD thesis of very high quality and I consider it to meet the highest international standards. I am thus recommending the admission of the candidate to the subsequent stages of the procedure, including the public defense.

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