University of Nottingham
UK | CHINA | MALAYSIA

October 25, 2022

Dr Sven Gnutzmann
Associate Professor
School of Mathematical Sciences
The University of Nottingham
University Park
Nottingham NG7 2RD
United Kingdom

Tel: +44 115 9514924
Fax: +44 115 9514951
sven.gnutzmann@nottingham.ac.uk

Report on the doctoral dissertation by Katarzyna Kowalczyk-Murynka

Calogero-Moser-Sutherland systems, quantization, topological methods and relationships with quantum chaos

Sven Gnutzmann (external assessor)

The topic of the thesis, the Calogero-Moser-Sutherland CMS) systems and their generalizations have been around for more than 50 years. As a paradigm model of non-trivial integrable many-body dynamics in one dimension they have found direct application in statistical physics, condensed matter physics and mathematical physics and many other fields. Some generalizations have surprising applications in seemingly distant fields such as integrable nonlinear partial differential equations or quantum chaos. The relation to quantum chaos is based on what is known in physics as the Pechukas-Yukawa gas – a generalization of CMS where the coupling coefficients are themselves dynamical variables that can be described in terms of a Hamiltonian approach. In spite of a thoroughly developed theory of CMS systems and their generalizations these systems continue to be a source of new ideas, developments and applications. In her dissertation thesis Ms Kowalczyk-Murynka develops new interesting insights into the algebraic and mathematical properties of a set of generalized classical and quantized CMS systems that are strongly related to the Pechukas-Yukawa gas and thus level-dynamics of perturbed systems. The latter are of interest in quantum chaos or disordered systems.

The dissertation thesis contains a number of interesting results:

1. Ms Kowalczyk-Murynka gives a constructive proof of the equivalence of different types of generalized CMS systems: one set where the dynamics of interactions are described by internal

degrees of freedom related to two particles (matrix approach) and another set where the dynamics is described in terms of internal degrees of freedom of the particles (vector approach). The detailed development of this result leads to several interesting insights into the mathematical structure of these models and as an aside lead to the discovery of a new CMS system with an interaction potential that is proportional to the inverse distance (rather than inverse squared distance in the known CMS systems).
This work has been published as a peer-reviewed research article in Physica D.

2. Ms Kowalczyk-Murynka describes in detail the dynamics of the internal degrees of freedom in generalized CMS systems. Using Lie methods she finds restrictions what values these parameters can take based on the initial phase space configuration. These results are new and interesting from a mathematical point of view as well as from possible applications in quantum chaos via level dynamics.
This work has been published as a preprint on ArXiv. My understanding (based on information from the supervisor) is that it is also submitted to a research journal and may lead to a further publication.

3. Ms Kowalczyk-Murynka develops quantization schemes for the generalized CMS systems that she considered in the classical part of her thesis. These results give interesting relations to other topics in theoretical and mathematical physics such as representation theory and general Lie Group actions in quantum systems.
I believe this work should be published as well in a research journal and my understanding (from the supervisor) is that this is planned.

In order to develop the proofs and the theory Ms Kowalczyk-Murynka has mastered a broad spectrum of background from theoretical and mathematical physics: Hamiltonian dynamics, integrability, symplectic reduction, Lie groups and Lie algebras along with their actions and representation theory, canonical quantization and more. In the dissertation thesis Ms Kowalczyk-Murynka explains non-trivial background and results in a mostly clear way. The few places which could be improved for clarity nonetheless show that she developed her results very independently.

Altogether I believe that this dissertation thesis has a high standard in terms of both content and presentation. If examination and defense are delivered on a similar standard I will recommend that she is awarded a PhD with a very good mark.

Nottingham, 25.October 2022

S. Gnutzmann

(Dr Sven Gnutzmann)