Opinion of the supervisor on the doctoral dissertation *Calogero-Moser-Sutherland systems, quantization, topological methods and relationships with quantum chaos* by Katarzyna Kowalczyk-Murynka

The Calogero-Sutherland-Moser systems introduced into research in the late 1960s and early 1970s, in both classical and quantum versions, are interesting cases of completely integrable multiparticle Hamiltonian systems with non-trivial, position-dependent interactions between particles. The existence of a sufficient number of constants of motion guarantees the complete integrability of these systems.. As it turns out, it is a consequence of the fact that the systems can be treated as reductions of linear systems in a suitably large phase space where the integration of equations of motion (finding action-angle variables) reduces to matrix diagonalization.

The configuration spaces of the Calogero-Moser and Sutherland-Moser systems are one-dimensional (straight line and circle, respectively). Thus, from the point of view of purely mechanical applications, they are of limited interest, but in applications to various physical issues (as discussed in the dissertation) they become interesting.

The doctoral dissertation of Ms. Katarzyna Kowalczyk-Murynka is devoted to generalizations of those systems in which the interactions between particles are dynamical - the bi-particle coupling "constants" of interactions between particles become additional dynamical variables with values depending on the current state of the system. The main motivation for the study of such systems, in addition to the interest in non-trivial Hamiltonian completely integrable systems, was the application to the description of the dynamics and statistical properties of quantum spectra of chaotic systems. Independently, systems were proposed in which the interactions between particles depended not on two-particle "coupling constants"("matrix" Calogero-Sutherland-Moser systems) but on some "internal states" of the interacting particles - e.g., spins (vector systems).

Interest in Calogero-Sutherland-Moser systems has not waned despite their half-century history, as evidenced by the literature cited in the dissertation. Despite the abundance of research, many detailed problems remain to be investigated. This is especially true of generalized systems. In her dissertation, Ms. Kowalczyk-Murynka examined many of them in a systematic manner. Her research was focused, foremost, on the analysis of the dynamics of the additional dynamical variables, i.e., the aforementioned couplings. For applications in the study of the properties of quantum Hamiltonian spectra, these couplings play a key role, as they affect the strength of "repulsion" between neighboring energy levels, which in turn, translates into differences in the statistical properties of spectral distributions allowing the detection of chaos and time symmetry of quantum systems. The second important research problem was the quantization of generalized Calogero-Sutherland-Moser systems, important for possible applications in the study of topological properties in condensed matter physics.

Thus, I consider the most important results of the dissertation to be

1. The analysis of the relationship between matrix and vector generalized Calogero-Sutherland-Moser systems, in particular showing their fundamental equivalence.
2. The discovery of a new generalized integral system of the generalized Calogero-Sutherland-Moser type, with an additional unexpected 1/r interaction.
3. Systematic analysis of the dynamics of couplings. The space of these variables has the natural structure of a Lie algebra, and the equations governing their dynamics are a natural generalization of the Euler equations with coefficients depending on the positions of the particles.
4. Investigation of the reachable sets of the dynamics of the couplings, which, as mentioned above, is of potential importance in the study of quantum spectra of chaotic systems.

5. Analysis of the quantization methods of classical generalized Calogero-Sutherland-Moser systems and showing the differences in the results, depending on the time symmetry of the system.

In her work, Ms. Kowalczyk-Murynka started with a geometric description of Hamiltonian dynamics, which required a very good knowledge of symplectic geometry and the theory of Lie groups and algebras. She also demonstrated excellent skills in analyzing differential equations and their use in the description of physical problems. Above all, however, she manifested remarkable independence and insight in the discovery, formulation and analysis of problems. The solutions she presented are characterized by very high mathematical precision and deep analysis. I have no hesitation in stating that the vast majority of the results obtained are completely independent solutions to many of the problems that she herself noticed and formulated. The role of the mentor was, first of all, to formulate general issues and point out possible directions of attack. The work resulted in a publication in Physica D on the analysis of matrix and vector generalized Calogero-Sutherland-Moser systems, and a preprint submitted to ArXiv, the contents of it and the relevant parts of the dissertation will be included in two forthcoming publications on (a) reachable sets of compressible dynamics and applications to quantum chaos theory, and (b) quantization of the considered systems.

Needless to say, therefore, I highly appreciate the results of Ms. Katarzyna Kowalczyk-Murynki and her dissertation, in which they were presented.

Marek Kuś
Professor of Physics,
Center for Theoretical Physics,
Polish Academy of Sciences

Warszawa, 22 August 2022