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Review of Doctoral Thesis "Quantum Universality and Membership Problems" presented by Msc. L. Mattioli, under the supervision of Prof. Adam Sawicki

The dissertation of the candidate Lorenzo Mattioli deals with the characterization of universal qudit gates, that is, investigating the necessary and sufficient conditions needed to ensure that a set of 1-qudit gates are universal. Universality in this context means that such a set S of gates should be able to generate any other 1-qudit gate in finite time with the desired approximation, or in other words, that can approximate any unitary operation by composing gates of the universal set. The work presented here is therefore a natural extension of the universality of 1-qubit gates, which can be constructed up to the desired accuracy by applying the Hadamard and the T-gate a finite number of times. As it is well known any universal 1-qubit gate and an entangling 2-qubit gates forms the universal gate set for quantum computation using qubits. A closely related problem, although different one, is membership, which again in this context refer whether one can approximate arbitrary well some gates independently if the gate set is universal.

The thesis is structured in several chapters leading to the reader towards the demonstration of the theorems which are the original research outputs of this thesis. The backbone of the dissertation relies on the one hand on the tensor symmetries needed for universality and membership in Hamiltonians, and such tensor symmetries lead in a natural way to approximate unitary t-designs. On the other hand, the concept of centralizer becomes a key point for dmonstrating universality. The path the author takes to demonstrated universality in SU(d) is complex and complete. First he deals with Lie Algebras, where the Universality and Membership problem can be reformulated properly. This detour is useful to solve the Membership and Universality problems from compact Lie Algebras, and see if the previous ideas of using the centralizer do help to solve the problem for SU(d).

There are several results obtained by the candidate what are remarkable, taking into account the difficulty and the required formalism to attack the problem of Universality for 1-qudit gates. First the candidate solve the problem of Membership problem for compact Lie algebras relying in the concept of a centralizer.

From there, the candidate is able to solve also the universality problem for the compact Lie Algebra, which follows as a consequence of the solution of the Membership for compact Lie algebras. These works share similarities with previous works by other authors but go clearly beyond these other works. In particular it is first shown that using the adjoint representation of a Lie algebra, the problem of membership and universality can be casted as a yes condition in terms of centralizers.

The most important theorem as far as I can understand is the fact that after a detour on checking membership and universality in compact Lie algebras, the use of tensor products extensions (similarly what it is use for symmetries in the universality of Hamiltonian sets) and centarlizers, the author has been able to constructively answer and provide an algorithm to solve the universality problem for SU(d), relying on t-designs, which are very important tools in the study of unitaries. In particular, he provides the delta-approximate t-design for both qubits (d=2) and qudits (d>2).
As a result, they provide a algorithmically friendly way to check if a given gate set is universal or not, which is much better of what is was provided before as certifying universality scales as d^4 for d>3 instead of the previous bound which scaled as d^[(d)^(5/2)]. Several examples are also given at the end of the thesis to see the relevance of this new bound.

Summarizing, the thesis reported by L. Mattioli represents a serious and well-grounded work on the universality of a set of gates acting on qudits. It demonstrates the maturarity of the candidate, as well as its mastering of the mathematical tools and concepts required to carry on this deep algebraic work, which is not at all trivial. The thesis is well structured in several chapters, each of them devoted to deal with one part of the mathematical structure needed to achieve the final results on universality. I have, however, missed in this thesis, a broader vision of which type of problems can be addressed with such techniques. Where and why will such a set of SU(d) gates implemented? there is a need to check universality? Which algorithms require qudit gates? How to extrapolate this universality in approaching unitaries to Hamiltonians? Can that construction be used to understand maybe Lindbladians, which do correspond to a unitary evolutions of a quantum system in contact with an environment? There are many questions concerning the role of unitaries that could be exploit maybe with such framework.

Final assessment *In my opinion the presented Dissertation has a clear scientific value and fulfills the requirements of a doctoral dissertation. The problem which is here considered is certainly difficult as well as important. Therefore I conclude that the presented dissertation meets the formal requirements for a Ph.D and recommend admission of the candidate to the subsequent stages of the procedure, including the public defense.*

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