Warsaw 3.12.2021

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Review of the doctoral thesis of Piotr Waluk
"On the problem of quasi-local mass in the weak field regime of gravity"

This thesis is devoted to the problem of energy in general relativity. As it is known from the begining of the Einstein theory it is not possible to define the local energy density which could be integrated over some domain. Only the total energy in an instant of time (or retarded time) in asymptotically flat spacetimes is well defined. In the case of a bounded domain V there are several proposals of so called quasi local mass in which energy is defined by properties of the 2-dimensional boundary of V. None of them is fully acceptable e.g. for some boundaries energy can be negative. Sections 2 and 3 of the thesis contain an excellent introduction to the problem. We see there not only definitions of energy but also many important comments on them. In particular in section 2 there is author's version of a proof of positivity of the ADM mass proposed by Kijowski in 1985. Section 3 starts dydacticly from a discussion of conservation laws, pseudotensors and Hamiltonian methods in general relativity. A role of boundary conditions is underlined. In the second part of this section there is a review and discussion of quasi local definitions of energy called after their inventors (Bartnik, Komar, Kijowski-Liu-Yau, Hawking).

Genuine new results are presented in sections 4 and 5. The main result is linking the Hawking mass to the Hamiltonian approach for metrics from a neighbourhood of the Kotler metric (also called the Schwarzschild-(anti) de Sitter metric). Section 4 contains a linearization of the Einstein equations written first in the 1+3 ADM formulation and then adapted to a foliation of the initial surface by spherical surfaces. The resulting 1+1+2 decomposition of the equations is given in terms of scalar functions on the spherical surfaces. A great advantage of this approach is that it reduces significantly the group of gauge transformations. Obtained results generalize those of Jezierski on perturbations of the Schwarzschild metric (ref. 36). They were published in a joint paper of Waluk and Jezierski (note this order of authors in ref. 79). Beside the equations transformations between new variables to those in the ADM approach are derived. The author also discusses a charge preservation related to invariant transformations and introduces the symplectic form and corresponding Hamiltonian defined up to boundary conditions.

A definition of a quasi local mass is higher rated if it can be obtained in the Hamiltonian approach. In section 5, based on a joint work of Kijowski, Jezierski and Waluk (ref. 42), the Hawking mas is compared with the Hamiltonian of the linearized theory from section 4. In the quadratic aproximation they differ by an angular momentum term and two boundary integrals. One of them is positive and the second one can be killed provided that the 2-dimensional surfaces in the 1+1+2 decomposition are so called rigid spheres (introduced by Gittel, Jezierski and Kijowski in ref. 29). Then the obtained relation implies positivity of the Hawking mass. Above results support an importance of the Hawking mass as well as an exceptional role of the rigid spheres among other surfaces with spherical topology.

Beside these main sections, in the thesis there are several appendices (together about 20 pages) to which some complicated technical derivations were moved. There is also Introduction (section 1) presenting basic material useful in further considerations. Everything is well writen and organized. The only discomfort for a reader is that a summary of the content and results in Introduction and Conclusions is too short. It seems that the author was already too exploited by cumbersome calculations and their description.

This thesis is exceptionally good concerning results and style. Mathematical operations certainly required a great effort and were difficult to present in a readable way. This task was successfully completed by the author. The thesis is well written and with a deep understanding of the subject (I had the same feeling when I was listening to talks of Waluk during seminars of our chair). The main results of the thesis (the linearization of the Einstein equations and the approximate equality of the Hawking mass and the invariant Hamiltonian) are already published in the highly rated journal Classical and Quantum Gravity. They were obtained in collaboration with both promotors of the thesis, but I know from them that Waluk played a crucial role in this collaboration. For the above reasons, in my opinion, the thesis is worth a distinction.

Therefore, I conclude that the presented dissertation meets the formal requirements for a Ph.D. thesis and recommend admission of the Candidate to the subsequent stages of the procedure, including the public defense.

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