Summary of PhD thesis "On the problem of quasi-local mass in the weak field regime of gravity"

The geometric nature of the field in general relativity makes it is impossible to define local energy density of gravity. Although a way has been discovered to define total energy (including the gravitational field contribution) of an isolated gravitating system, its application requires a rather restrictive set of conditions to be fulfilled. This has motivated researchers to look for a method of assigning an energy value to an extended but finite region of spacetime — a "quasi-local" mass — hopefully under less restrictive assumptions. In spite of the amount of proposed constructions, none has yet been proven to be a fully satisfactory candidate.

This dissertation describes some of the issues connected with exploring the concept of energy and mass within the scope of general relativity and methods used to tackle these problems. The definition of the total mass of a gravitating system (the ADM mass) is briefly presented as an established point of reference. It is accompanied by a sketch of a simple proof of its positivity, based on modifying the so-called scalar constraint with appropriate gauge conditions.

This is followed by a discussion of the mathematical apparatus applied in attempts at defining quasi-local energy of gravity. Special attention is given to the problem of boundary conditions imposed on the edge of the extended region. They play a crucial role in defining the energy content, since they not only govern the way in which the region in question interacts with the "outside", but also provide a reference frame — a necessary component to give the term "energy" a precise meaning. The discussion is illustrated with several examples of proposed quasi-local mass constructions.

The linear approximation of general relativity is described in detail next. This is done with the help of a gauge invariant formalism, which the author helped develop. In this approximation, the problem of energy is well understood and an explicit Hamiltonian function can be constructed in a canonical way from the symplectic form of the theory.

Finally, a viability criterion for quasi-local mass construction is discussed: a sensible energy expression for the full non-linear theory should be approximated by the Hamiltonian function of linear gravity. In particular, the ADM mass satisfies this condition. As a most recent result of the author, obtained in collaboration with his advisors, this criterion has been shown to be fulfilled by the Hawking quasi-local mass, under certain conditions. Some of these conditions can be interpreted as a restriction on the shape of the boundary surface, an approximated form of the so-called rigid sphere conditions.