This brief document serves as a report on the thesis *Quantum mappings and designs* by Grzegorz-Rajchel Mieldzoc at the Centrum Fizyki Teoretycznej Polskiej Akademii Nauk.

The thesis is a collection of results at the interplay of quantum information science, mathematical physics, and matrix theory – combinatorial matrix theory. It includes observations of combinatorial nature and linear algebraic ideas. It includes works on several types of quantum designs, as latin squares and generalizations. The volume of scientific publications coming out form the thesis work is remarkable and the candidate has done an amazing job in putting toghether a number of different scientific results but with a single guiding theme.

When reading the thesis, I got the impression that a number of the results are limited in scope because restricted to special cases. But then, this is the price to pay when working on areas like design theory. It's extremely hard to get results that can generalize for many dimensions for example, and the work done in this thesis is remarkable, in boths technical depth and creativity.

The unistochasticity problem is far from being well-understood. Chapter 2.
Definition 25 (Bracelet matrices). I'd be curious to see what is the combinatorial version of this. I have been interested in unistochasticity and graphs while doing my PhD. Can the notion of a bracelet matrix translate into something generally applicable to (0,1)-matrices? The application of rays inside the Birkhoff polytope in 2.8 is interesting.

Chapter 3 jumps onto a different problem, related to entangling power. Comments: can we move Fig. 3.1 further down the section? Not much work has been done on the multipartite case so far and the contribution here is valuable. I like Theorem 48. Interesting combinatorial flavor. Question: I'd be curious to see what happens when we restrict the unitaries to permutations only – for the bipartite case, see my work with Sudbery, Gosh, et al. – I suppose 2005.

In page 43, "Therefore, we are tempted to believe that our results will prove useful for benchmarking quantum devices, with a possible extension into the domain of entanglement creation." I personally don't see how and I think that this is a stretch. The beauty of working on this problem is more of mathematical nature. Running some computation on a quantum device has a lot to do with the architecture of the device, e.g., it's connectivity. The results of this chapter don't take this minimally into account and I believe that it's not important to make a remark about this potential use – which is unlikely, but nonetheless this does not diminish the result itself. In other word, the candidate should not worry if the result does not find an application now, but should also not venture into creating such a weak connection with an unrelated topic in device physics just because it's fashionable – I leave this to the author's jusdgement.

In Chapter 4, Lemma 56 and Definition 57 are interesting. They open up a number of ideas. The connection with Hessians in 4.14 is nontrivial. The candidate manages to solve a complex combinatorial problem via what he calls golden AME(4,6) state. I like this chapter a lot. It's a tour de force and Fig. 4.12 shows somehow self-evidently the intricacy of getting his result.

In Chapter 5, the connections between the SudoQ game and MUBs is interesting and worthy of further attention. I personally would love to see a full characterization of the cardinality of quantum Latin squares. This is an open problem of and I hope that the author and/or the community will further work on this.

I really enjoyed reading this work. **I am fully confident that the thesis satisfies the high international standards for a PhD degree** and I wish the candidate a successful career.

Thanks for this opportunity. For any question please don't hesitate to reach out to me via my email address s.severini@ucl.ac.uk.

Simone

Prof Simone Severini
Professor of Physics of Information
Department of Computer Science, University College London (UCL)
London, UK