Report on the thesis

Quantum mappings and designs

by Grzegorz Rajchel-Mieldzioć

The thesis of Grzegorz Rajchel-Mieldzioć submitted in September 2021 to the Center for Theoretical Physics of the Polish Academy of Science deals with fundamental questions in the theory of *quantum information*. The thesis consists of six chapters and seven appendices, dealing with the more technical issues discussed in the main text. The first part of the thesis is focused on *quantum mappings*, where unistochastic matrices and the entangling power of quantum gates is discussed. The second part of the thesis is about *quantum designs*, where absolutely maximally entangled states and SudoQ grids are discussed.

In the first chapter of the thesis, the author first reviews the *basics of quantum information theory*, with emphasis on the linear algebraic aspects. The notion of entanglement is discussed at length, for states, as well as for quantum gates. In the final section of this introductory chapter, the author presents in detail his own contributions. This section is very well written and allows the reader to precisely identify what are the results obtain in the series of papers the thesis is based on, as well as the author's own contribution towards obtaining these results.

The second chapter is concerned with the theory of *unistochastic matrices*. These are bistochastic matrices which come from unitary matrices by taking the squared absolute values of the entries. Unistochastic matrices of size 3 have been characterized by the so-called bracelet conditions, and very few results exist for superior dimensions. The contribution of the author to this topic is a numerical algorithm for checking unistochasticity for matrices of dimension 4. Then, a series of theoretical results regarding special classes of unistochastic matrices is presented. Importantly, the author shows that circulant bistochastic matrices are unistochastic if and only if they satisfy the bracelet conditions.

The third chapter discusses the *entangling power* of unitary gates acting on multipartite systems. This work is a generalization of the bipartite case, and the techniques used here (orthogonal Weingarten calculus) are very interesting. The author computes the average entangling power of a random gate acting on the tensor product of three Hilbert spaces using, as a measure of multipartite entanglement, the one-tangle.

The fourth chapter is closely related to the third one, in the sense that here the focus is on the optimization of the entangling power considered previously. The focus is put here on the only case left open in the literature for four parties, that of four 6-dimensional quantum systems. The relation to *absolutely maximally entangled states* is presented, and a solution is found by rounding off the output of a numerical search algorithm. This constitutes what is, to my mind, the most impressive result of this PhD thesis, the construction of an AME$(4, 6)$ state. The relation between entangling power, AME states, and (orthogonal) quantum Latin squares is clearly presented,

as is the search for orthogonal quantum Latin squares of order 6. The chapter culminates with the description of such a square, called the golden AME$(4, 6)$ state.

The fifth chapter is about a quantum generalization of Sudoku puzzles, called *SudoQ*. The framework is that of quantum Latin squares, with additional sub-square constraints, as for classical Sudoku. The author constructs quantum grids of maximal cardinality and connects SudoQ grids to mutually unbiased bases. The results in this chapter add to the understanding of both low-dimensional and general SudoQ grids.

Ther last chapter of the thesis contains a detailed list of open problems left unresolved by the author and which are in direct relation with the topics discussed previously. This last chapter shows that the candidate has the scientific maturity needed to recognize the important questions which stem out of his PhD work.

The PhD thesis of Grzegorz Rajchel-Mieldzioć contains results about quantum states and quantum gates having particular properties, mostly related to entanglement, which sets them apart from purely classical objects. It is the search for mappings and designs which are, in some sense, the "most quantum" which can be seen as the main motivation of the candidate's PhD work. In the era of the first quantum computers, understanding the limits of quantum protocols, and of the advantages they might have over classical protocols, is of paramount importance. For this reason, I consider that the PhD thesis *Quantum mappings and designs* has great scientific merit, in the field of theoretical quantum information theory.

The thesis is very well written, and the notions presented are often described using figures and photographs. The presentation of the topics and of the results is very well done, reading the thesis was a pleasure. I would also like to emphasize the very nice figures and photographs in the text, which add a lot to the quality of the thesis.

The decision to split the thesis into a main part, followed by a series of technical appendices containing the full text of the scientific papers is, in my mind a good one. Sometimes, details are lacking in the main text, but the reader can find them in the appendices. For example, more detailed analysis of the algorithm for checking membership in the set of unistochastic matrices from Section 2 would have been interesting. Similarly, the proof of Remark 67, regarding the non-existence of quantum Latin squares of size 3 is a bit fast, given that the result is non-trivial.

Overall, I find the PhD thesis of Grzegorz Rajchel-Mieldzioć of very high quality. The author has published 5 scientific papers during this thesis, all containing major contributions to the field of quantum information theory. I consider that the thesis meets the highest international standards for a PhD degree. I am thus recommending to proceed further with awarding Grzegorz Rajchel-Mieldzioć a PhD degree.

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