The Standard Model (part I)

Speaker

- Jens Kunstmann
- Student of Physics in 5<sup>th</sup> year at Greifswald University, Germany

Location

- Sommerakademie der Studienstiftung, Kreisau 2002
Topics

- Introduction
- The fundamental interactions and the scope of the Standard Model
- Particles in Physics
- Elementary Particles
- Composite Particles
- Quantum Numbers and Conservation Laws
- Feynman Diagrams

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atom

nucleus

proton
The four Interactions

**Strong**
- bonding inside an atomic nucleus
- bonding of quarks to nucleons
- Strength: 1

**Electromagnetic**
- forces between charges
- electricity, electronics
- electromagnetic radiation
- chemical bonding
- Strength: $10^{-2}$

**Weak**
- decay processes of nuclei and particles
- Strength: $10^{-14}$

**Gravitational**
- forces between masses
- Strength: $10^{-38}$

Standard Model in terms of (special) relativistic Quantum Mechanics

Don’t like each other

General Relativity
The classical picture of a particle is a mass point that at the time \( t_0 \) is at position \( \vec{x} \) with a momentum \( \vec{p} = m \cdot \vec{v} \).

This picture is not longer adequate in quantum mechanics where according to the **Heisenberg Uncertainty Principle** we can’t simultaneously determine the position and the momentum of a particle with arbitrary precision: \( \Delta x \cdot \Delta p_x \geq \hbar \).

To confuse you even more, there are also particles that don’t have a mass!

Furthermore all microparticles have a **Spin**, a property that has no classical analogue.

To take all this into account we use an abstract description where the **state** of a particle is determined by a set of **Quantum Numbers**.
Particles in Physics:

The Spin

- The Spin $\vec{S}$ of a particle is an angular momentum in its own rest frame.
- Beside position or momentum it is an additional degree of freedom for every particle and must be specified to determine its state.
- Angular momenta in Quantum Mechanics are characterized by two quantum numbers $S$ and $S_3$.
  - $S$ is connected to the absolute value of the angular momentum vector: $|\vec{S}| = \sqrt{S(S+1)}\hbar$.
  - Allowed values for $S$ are integer and half-integer numbers: $0, 1/2, 1, 3/2, ...$.
  - $S_3$ determines the directions of $\vec{S}$ according to a quantization axis $x_3$.
  - Possible values are $S_3 = -S, -S + 1, ..., 0, ..., S - 1, S$.
Every particle has a fixed value for $S$ associated to it!

Particles with half-integer spin are called **Fermions**

Particles with integer spin are called **Bosons**

Example: Electron $S = \frac{1}{2}$

\[
\{ \begin{array}{l}
\frac{1}{2} \quad \text{― "Spin up"}\\
-\frac{1}{2} \quad \text{― "Spin down"}
\end{array}
\]
Particles in Physics:

**Antiparticles**

- For every **charged** particle there is an associated **antiparticle**, it has the same mass and spin (isospin, lifetime) but opposite charge (baryon #, lepton #, parity, quark flavours).
- Some **uncharged** particles have antiparticles others don’t! (or we simply can’t distinguish between them).
- If a particle’s symbol is e.g. $p$, its antiparticle’s symbol is $\bar{p}$.
- **Examples:**
  - Electron ($e^-$) $\leftrightarrow$ Positron ($e^+$)
  - Neutron ($n$) $\leftrightarrow$ Antineutron ($\bar{n}$)
  - Photon (no antiparticle: $\times$)
  - Antiphoton (no antiparticle: $\times$)

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In the Standard Model there are three groups of elementary particles:

- **Spin−1/2−fermions**
  - Quarks
  - Leptons

- **Spin−1−Bosons**
  - Gauge−Bosons
Elementary Particles:

### Quarks & Leptons

<table>
<thead>
<tr>
<th>Generation</th>
<th>Quarks</th>
<th>Leptons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( u ) (up)</td>
<td>( e^- ) (electron)</td>
</tr>
<tr>
<td></td>
<td>( \overline{u} ) (down)</td>
<td>( \nu_e ) (electron−neutrino)</td>
</tr>
<tr>
<td>2</td>
<td>( c ) (charm)</td>
<td>( \mu^- ) (muon)</td>
</tr>
<tr>
<td></td>
<td>( \overline{c} ) (strange)</td>
<td>( \nu_\mu ) (muon−neutrino)</td>
</tr>
<tr>
<td>3</td>
<td>( t ) (top)</td>
<td>( \tau^- ) (tauon)</td>
</tr>
<tr>
<td></td>
<td>( \overline{t} ) (bottom)</td>
<td>( \nu_\tau ) (tauon−neutrino)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>charge</th>
<th>+2/3</th>
<th>-1/3</th>
</tr>
</thead>
</table>

- the name of a quark is called its **flavour**
- each quark comes in three **colours**: \( r, g, b \)
- each anti–quark comes in „anticolours“: \( \bar{r}, \bar{g}, \bar{b} \)
- for each lepton there is an antilepton
- the neutrinos are massless in SM

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## Gauge Bosons

In SM interactions are mediated by the exchange of virtual gauge bosons = „force–carrier particles“

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Boson</th>
<th>Properties of Bosons</th>
<th>Range of interaction</th>
<th>Acts between</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electromagnetic</td>
<td>$\gamma$</td>
<td>massless</td>
<td>$\infty$</td>
<td>electric charges</td>
</tr>
<tr>
<td>Weak</td>
<td>$W^+, W^-$</td>
<td>charged/uncharged/massive</td>
<td>$\approx 2 \times 10^{-18}$ m</td>
<td>all particles</td>
</tr>
<tr>
<td>Strong</td>
<td>$g$</td>
<td>massless/fundamental: $\infty$; residual: $\approx 10^{-15}$ m</td>
<td>colour-charges</td>
<td></td>
</tr>
</tbody>
</table>
Composite Particles

- Free quarks have never been observed
- The reason is that the strong interaction only permits free particles that have no net colour!
- The fundamental strong interaction binds quarks together to form:

**Hadrons**

- **Baryons/Anti-Baryons**
  - $q, q_g q_b$
  - $\bar{q}_r \bar{q}_g \bar{q}_b$

- **Mesons**
  - $q_c \bar{q}_c$

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Baryons are Fermions

\[ \begin{array}{c|c|c}
\text{Spin} & \text{uud} & \text{udd} \\
\hline
\uparrow \uparrow \uparrow & 1/2 & \text{proton (p)} \quad \text{neutron (n)} \\
\hline
\uparrow \uparrow \uparrow & 3/2 & \Delta^+ \quad \Delta^0 \\
\end{array} \]

Mesons are Bosons

\[ \begin{array}{c|c|c|c}
\text{Spin} & \text{u\bar{d}} & \text{u\bar{u}} & \text{d\bar{u}} \\
\hline
\uparrow \downarrow & 0 & \pi^+ & \pi^0 & \pi^- \\
\hline
\uparrow \uparrow & 1 & \rho^+ & \rho^0 & \rho^- \\
\end{array} \]

Hadrons interact by the exchange of Mesons

This is the **residual strong interaction** which is very short-ranged

\[ \approx 10^{-15} \text{ m} \]
Composite Particles:

The Atom
Particle Summary

Particles

Fermions
- fundamental
  - Quarks: u, d, c, s, t, b
  - Leptons: \( e^-, \mu^-, \tau^- \), \( \nu_e, \nu_\mu, \nu_\tau \)

- composite
  - Baryons: proton, neutron, etc.
  - leptons: \( \nu_e, \nu_\mu, \nu_\tau \)

Bosons
- fundamental
  - Gauge-Bosons: \( \gamma, W^+, W^-, Z^0, g \)

- composite
  - Mesons: pions, rhos, etc.

Gauge-Bosons
- point-like, Spin = 1/2
- extended particles, Spin = 1/2, 3/2, ...
- pointlike, Spin = 1
- Spin = 1, 2, ...

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Quantum Numbers

• quantum numbers (QN) define the state of a particle
• a complete set of QN gives a unique description of a quantum state of a particle
• we already introduced quark flavours and QN for spin ($S, S_3$) and colour
• for composite particles the quantum numbers of the constituents (except parity and charge conjugation) just add.
• physicists always look for „good“ QN, these are QN that are conserved in interactions
Quantum Numbers:

The Lepton Numbers

<table>
<thead>
<tr>
<th></th>
<th>e</th>
<th>ve</th>
<th>μ</th>
<th>νμ</th>
<th>τ</th>
<th>ντ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron #: L_e</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>muon #: L_μ</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>tauon #: L_τ</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

- Antileptons have negative lepton numbers
- The number of leptons of each generation is conserved in all interactions

muon−decay:

\[ \mu^- \rightarrow e^- + \bar{v}_e + v_\mu \]

\[ L_\mu = 1 \rightarrow L_\mu = 1 \]
\[ L_e = 0 \]
\[ L_e = 1 - 1 = 0 \]

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Quantum Numbers:

The Baryon Number

- Baryons: $B = 1$
- Anti-Baryons: $B = -1$
- Others: $B = 0$

- The number of Baryons is conserved in all interactions

  it follows:

  - Quarks: $B = 1/3$
  - Antiquarks: $B = -1/3$
Quantum Numbers:

**Isospin**

- All quarks have very different masses, except the \( u \) and \( d \), their masses are quite equal \( \rightarrow \) some symmetry

- We expect hadrons made up of \( u \) and \( d \) quarks to exist in **groups**

- The members of a group have the same strong interaction (same spin, baryon #, flavour and parity) but different electric charges

- We use spin algebra to describe these groups by using the quantum numbers \( I \) and \( I_3 \)
Quantum Numbers:

**Isospin**

- $I$ is the group label and $I_3$ is the particle label

**Examples:**

1. The up and down quarks form an Isospin-$1/2$ group.
   - If $I = \frac{1}{2}$ then $I_3 = \pm \frac{1}{2}, +\frac{1}{2}$
   - Quarks: $u$, $d$

2. The proton and neutron also form an Isospin-$1/2$ group.
   - $I_3 = +1/2, -1/2$
   - Nucleons: $p$, $n$

3. The three pions form an Isospin-$1$ group.
   - If $I = 1$ then $I_3 = -1, 0, 1$
   - Pions: $\pi^+(ud)$, $\pi^0(uu)$, $\pi^-(du)$

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Quantum Numbers:

The Flavours

- For quarks of the 2\textsuperscript{nd} and 3\textsuperscript{rd} generation the Isospin symmetry does not apply anymore.

- Here we have a flavour QN for each quark:

<table>
<thead>
<tr>
<th></th>
<th>u</th>
<th>d</th>
<th>c</th>
<th>s</th>
<th>t</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$</td>
<td>1/2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$I_3$</td>
<td>1/2</td>
<td>-1/2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$C$ (charm)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$S$ (strangeness)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$T$ (truth)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$B$ (beauty)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

- Antiparticles have the same QN with opposite sign.
Quantum Numbers:

**Hypercharge**

- Hypercharge $Y$ is the sum of Baryon Number $B$ and the flavours $C, S, T, B$:

$$Y = B + C + S + T + \tilde{B}$$

- Hypercharge $Y$ and Isospin $I_3$ allow to calculate the electric charge $Q$ of a particle:

$$Q = I_3 + Y/2$$
Quantum Numbers:

Example

- Beta-decay:

\[ n \rightarrow p + e^- + \bar{\nu}_e \]

- Charges:
  - \( Q = 0 \) → \( Q = +1 \)
  - \( -1 = 0 \)

- Baryon Number (B) = 1:
  - \( B = 1 \)

- Lepton Number (\( L_e \)) = 0:
  - \( L_e = +1 \)
  - \( -1 = 0 \)

- Isospin (\( I \)) = 1/2:
  - \( I = 1/2 \)

- Isospin (\( I_3 \)) = -1/2:
  - \( I_3 = +1/2 \)

- All Quantum Numbers are conserved except Isospin \( I_3 \)
### Quantum Numbers:

#### Conservation Laws

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Strong</th>
<th>Electromagnetic</th>
<th>Weak</th>
</tr>
</thead>
<tbody>
<tr>
<td>energy</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>momentum</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>angular momentum</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>charge</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>lepton number</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>baryon number</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Isospin $I_3$</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>Flavour</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>Parity</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>charge conjugation</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
</tbody>
</table>

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Feynman Diagrams

- Interaction processes are represented by Feynman diagrams:
  - e.g.: electron scattering: \( e^- + e^- \rightarrow e^- + e^- \)

  ![Feynman Diagrams](image)

  2 vertices  4 vertices

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td></td>
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<tr>
<td></td>
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</tr>
</tbody>
</table>

  - particles
  - antiparticles
  - virtual bosons

  - Time runs from left to right

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Feynman Diagrams:

Virtual Processes

- Feynman Diagrams are made up from a combination of basic processes / basic vertices, e.g.:
Feynman Diagrams:

Virtual Processes

- All virtual processes of a family have the same probability to occur.
- A measure of this probability is the Coupling Constant $g$
- Electromagnetic: $g = \alpha$
- Weak: $g = g_w$
- By combining two or more virtual processes we get a real process.
- The Order $n$ of a process is equal to the number of vertices.
- The higher the order of a process the lower is the probability for it to occur! $W \sim g^n$
- $\Rightarrow$ We can neglect higher order processes in most cases.
Feynman Diagrams:

Virtual Processes

- They are called „virtual“ because they violate energy and momentum conservation!
- In quantum mechanics such a violation is possible (!), but only for a short period of time:
  \[ \tau \cdot \Delta E \geq \hbar \] (Uncertainty Principle for energy and time)
- This means that a massive \((E = mc^2)\) virtual boson can only exist for a certain length of time and can only „travel“ a certain range then
- That’s why interactions that exchange massive bosons (weak and residual strong interaction) are always short-ranged
- Although energy conservation is violated at the first vertex, this can be compensated by a similar violation at the second vertex to give exact energy conservation overall \(\rightarrow \) real process
- The initial and final state conserve energy and momentum
Feynman Diagrams:

Example

\[ \beta\text{-decay: } n \rightarrow p + e^- + \bar{\nu}_e \]
End of Part I