LONG-LIVED QUANTUM MEMORY USING NUCLEAR SPINS

Laboratoire Kastler Brossel

A. Sinatra, G. Reinaudi, F. Laloë (ENS, Paris)

A. Dantan, E. Giacobino, M. Pinard (UPMC, Paris)
NUCLEAR SPINS HAVE LONG RELAXATION TIMES

Ground state He3 has a purely nuclear spin $\frac{1}{2}$

Nuclear magnetic moments are small
$\rightarrow$ Weak magnetic couplings
$\rightarrow$ dipole-dipole interaction contributes for $T_1 \approx 10^9$ s (1mbar)

Spin 1/2 $\rightarrow$ No electric quadrupole moment coupling

In practice
$T_1$ relaxation can reach 5 days in cesium-cotated cells
and 14 days in sol-gel coated cells
$T_2$ is usually limited by magnetic field inhomogeneity
Examples of Relaxation times in $^3$He

$T_1 = 344 \text{ h}$

$T_2 = 1.33 \text{ h}$

Ming F. Hsu et al.
Sol-gel coated glass cells

Magnetometer with polarized $^3$He
Quantum information with continuous variables

Spin Squeezed atoms
Quantum memory
SQUEEZING OF LIGHT AND SQUEEZING OF SPINS

One mode of the EM field

\[ Y = \text{i}(a^\dagger - a) \]

\[ X = (a + a^\dagger) \]

Coherent state \( \Delta X = \Delta Y = 1 \)

Squeezed state \( \Delta X > 1 \)
\[ \Delta Y < 1 \]

N spins 1/2

Coherent spin state CSS (uncorrelated spins)

\[ \Delta S_x = \Delta S_y = |<S_z>|/2 \]

Projection noise in atomic clocks

Squeezed state

\[ \Delta S_x^2 < N/4 \]
\[ \Delta S_y^2 > N/4 \]
How to access the ground state of $^3$He?

During a collision the metastable and the ground state atoms exchange their electronic variables.

Colegrove, Schearer, Walters (1963)
Typical parameters for 3He optical pumping

An optical pumping cell filled with ≈ mbar pure $^3$He

**Metastables:**
\[ n = 10^{10-10^{11}} \text{ at/cm}^3 \]

**Metastable / ground:**
\[ \frac{n}{N} = 10^{-6} \]

**Laser Power for optical pumping**
\[ P_L = 5 \text{ W} \]

The ground state gas can be polarized to 85% in good conditions.
A SIMPLIFIED MODEL: SPIN \( \frac{1}{2} \) METASTABLE STATE

\[
S = \sum_{i=1}^{n} s_i, \quad I = \sum_{i=1}^{N} i_i
\]

Partridge and Series (1966)

\[
\frac{d\langle S \rangle}{dt} = -\gamma_m \langle S \rangle + \gamma_g \langle I \rangle \\
\frac{d\langle I \rangle}{dt} = -\gamma_g \langle I \rangle + \gamma_m \langle S \rangle
\]

Heisenberg Langevin equations

\[
\frac{dS}{dt} = -\gamma_m S + \gamma_g I + f_s \\
\frac{dI}{dt} = -\gamma_g I + \gamma_m S + f_i
\]

Rates

\[
\frac{\gamma_m}{\gamma_g} = \frac{N}{n}
\]

Langevin forces

\[
\langle f_a(t) f_b(t') \rangle = D_{ab} \delta(t-t')
\]
Atoms prepared in the fully polarized CSS by optical pumping

Spin quadratures

\[ S_x = \frac{(S_{21} + S_{12})}{2} \quad ; \quad S_y = i \left( S_{12} - S_{21} \right)/2 \]

This state is **stationary** for metastability exchange collisions.
SQUEEZING TRANSFER TO FROM FIELD TO ATOMS

Raman configuration $\Delta > \gamma, \delta$

Control classical field of Rabi frequency $\Omega$

Exchange collisions

A, $A^\dagger$ cavity mode

$squeezed$ $vaccum$

$H = g (S_{32} A + h.c.)$

Linearization of optical Bloch equations for quantum fluctuations around the $fully$ $polarized$ initial state
Linear coupling for quantum fluctuations between the cavity field and metastable atoms coherence $S_{21}$

Adiabatic elimination of $S_{23}$

\[
\frac{d}{dt} S_{21} = i \left( \delta + \frac{\Omega^2}{\Delta} \right) S_{21} + \frac{\Omega g n}{\Delta} A
\]

Resonant coupling condition: $\delta_{\text{shift}} = 0$

$\delta = (E_2 - E_1) - (\omega_2 - \omega_1)$

Two photon detuning

The metastable coherence $S_{12}$ evolves at the frequency $(\omega_1 - \omega_2)$.

Exchange collisions give a linear coupling between $S_{12}$ and $I_{\uparrow \downarrow}$.

Resonant coupling condition:

$$E_{\uparrow} - E_{\downarrow} = (\omega_1 - \omega_2)$$

Resonant coupling in both metastable and ground state is possible in a magnetic field such that the Zeeman effect in the metastable compensates the light shift of level 1.
SQUEEZING TRANSFER TO GROUND STATE ATOMS

Zeeman effect:

\[ E_2 - E_1 = 1.8 \text{ MHz/G} \]

\[ E_{\uparrow} - E_{\downarrow} = 3.2 \text{ kHz/G} \]

Resonance conditions in a magnetic field

\[ E_2 - E_1 + \frac{\Omega^2}{\Delta} = E_{\uparrow} - E_{\downarrow} = (\omega_1 - \omega_2) \]
When polarization and cavity field are adiabatically eliminated

\[
\Delta I_y^2 = \frac{N}{4} \left[ 1 - \frac{\gamma_m}{\Gamma + \gamma_m} (1 - \Delta A_x^{in^2}) \right]
\]

\[
\Delta S_y^2 = \frac{n}{4} \left[ 1 - \frac{\Gamma}{\Gamma + \gamma_m} (1 - \Delta A_x^{in^2}) \right]
\]

**Pumping parameter**

\[
\Gamma = \frac{\gamma \Omega^2}{\Delta^2} (1 + C)
\]

**Cooperativity**

\[
C = \frac{g^2 n}{K \gamma} \approx 100
\]

**Strong pumping**  \( \Gamma >> \gamma_m \)  the squeezing goes to metastables

**Slow pumping**  \( \Gamma << \gamma_m \)  the squeezing goes to ground state
(1 − ΔA_{x}^{in^2}) = 0.5 \quad C = 500
Exchange collisions and correlations

\[ <S^2> = \sum_{i=1}^{n} <s_i^2> + \sum_{i \neq j}^{n} <s_i s_j> = \frac{n}{4} + n(n + 1) <s_i s_j> \]

\[ <I^2> = \sum_{i=1}^{N} <i_i^2> + \sum_{i \neq j}^{n} <i_i i_j> = \frac{N}{4} + N(N + 1) <i_i i_j> \]

Exchange collisions tend to equalize the correlation function

When exchange is dominant \((\gamma_m >> \Gamma)\), and for \(n, N >> 1\)

\[ \left( \frac{\Delta I^2}{N/4} - 1 \right) = \frac{N}{n} \left( \frac{\Delta S^2}{n/4} - 1 \right) \]

A weak squeezing in metastable maintains strong squeezing in the ground state
Build-up of the spin squeezing in the metastable state

\[ < S_y^2 > (t) - < S_y^2 >_s = 0.55 \exp(-2\Gamma t) \]

\[ \Gamma = 10 \gamma_m = 5 \times 10^7 \text{ s}^{-1} \]

Build-up of the spin squeezing in the ground state

\[ < I_y^2 > (t) - < I_y^2 >_s = 0.55 \exp(-2\Gamma_g t) \]

\[ \Gamma_g = \frac{2\Gamma \gamma_g}{\gamma_m + \Gamma} \approx 1 \text{ s}^{-1} \]
The writing time is limited by $\gamma_g \propto n$ (density of metastables)
By lighting up only the coherent control field $\Omega$, in the same conditions as for writing, one retrieves transient squeezing in the output field from the cavity.
READ OUT OF THE NUCLEAR SPIN MEMORY

\[ E_{LO}(t) = \exp(-\Gamma_g t) \]

Temporally matched local oscillator

\[ P(t_0) = \int_{\Delta\omega} \frac{d\omega}{\Delta T} \int_{t_0}^{t_0 + \Delta T} dt \ e^{-i\omega(t-t')} E_{LO}(t) E_{LO}(t') \langle A_{out}(t) A_{out}(t') \rangle \]

Read-out function for \( \Delta T > 1/\Gamma_g \)

\[ R_{out}(t_0) = 1 - \frac{P(t_0)/\Delta\omega}{\left[P(t_0)/\Delta\omega\right]_{CCS}} = (1 - \Delta I_y^2) \exp(-2\Gamma_g t_0) \]
Real atomic structure of 3He

\[ \dot{\rho}_g = \gamma_g (-\rho_g + Tr_e \rho_m) \]

\[ \dot{\rho}_m = \gamma_m (-\rho_m + \rho_g \otimes Tr_n \rho_m) \]
Real atomic structure of 3He

\[ 2^3P_0 \quad 2^3S_1 \quad 1^1S_0 \]

Lifetime of metastable state coherence: \( \gamma_0 = 10^3 \text{s}^{-1} \) (1torr)

Exchange Collisions

\[ \Omega \quad \Omega \quad A \quad A \]

\[ F=1/2 \quad F=3/2 \]
We find the **same** analytic expressions as for the simple model if the field and optical coherences are adiabatically eliminated.

\[ \gamma_0 \approx \Gamma \]

Adiabatic elimination not justified

\[ \Gamma \approx \gamma \]

Effect of

\[ \gamma_m = 5 \times 10^6, \quad \gamma = 2 \times 10^7, \quad \gamma_0 = 10^3, \quad \kappa = 100\gamma, \quad C = 500, \quad \Delta = -2 \times 10^3 \]
EFFECT OF A FREQUENCY MISMATCH IN GROUND STATE

We have assumed resonance conditions in a magnetic field

\[ E_4 - E_3 + \frac{\Omega^2}{\Delta} = E_{\uparrow} - E_{\downarrow} = (\omega_1 - \omega_2) \]

\[ E_4 - E_3 \approx 1.8 \text{ MHz/G} \]
\[ E_{\uparrow} - E_{\downarrow} = 3.2 \text{ kHz/G} \]

What is the effect of a magnetic field inhomogeneity?

De-phasing between the squeezed field and the ground state coherence during the squeezing build-up time
Example of working point:

\[ \Gamma = 0.1 \gamma_m, \quad B = 57 \text{ mG}, \quad \text{Larmor} = 184 \text{ Hz} \]
LONG-LIVED NON LOCAL CORRELATIONS BETWEEN TWO MACROSCOPIC SAMPLES

Julsgaard et al., Nature (2001) : $10^{12}$ atoms  $\tau=0.5$ms

Correlated macroscopic spins ($10^{18}$ atoms)?
Two modes of the EM field

**Quadratures**

\[ A_{x1(2)} = (A_{1(2)} + A_{1(2)}^+) \quad A_{y1(2)} = i(A_{1(2)}^+ - A_{1(2)}) \]

\[
\Im F = \frac{1}{2} \left[ \Delta^2 (A_{x1} - A_{x2}) + \Delta^2 (A_{y1} + A_{y2}) \right] < 2
\]

Two collective spins

\[ \langle I_{z1} \rangle = \langle I_{z2} \rangle \approx \frac{N}{2} \]

\[
\Im_A = \frac{2}{N} \left[ \Delta^2 (I_{y1} - I_{y2}) + \Delta^2 (I_{x1} + I_{x2}) \right] < 2
\]
LONG-LIVED NON LOCAL CORRELATIONS BETWEEN TWO MACROSCOPIC SAMPLES

\[
\begin{align*}
\left(\Gamma_g - i\omega\right) I_{y1(2)} &= \beta A^{in}_{x1(2)} + \text{noise} \\
\left(\Gamma_g - i\omega\right) I_{x1(2)} &= -\beta A^{in}_{y1(2)} + \text{noise}
\end{align*}
\]
LONG-LIVED NON LOCAL CORRELATIONS BETWEEN TWO MACROSCOPIC SAMPLES

In terms of inseparability criterion of Duan and Simon

\[ \mathcal{I}_A = \frac{\gamma_m}{\gamma_m + \Gamma} \mathcal{I}_F + 2 \left( \frac{\Gamma}{\gamma_m + \Gamma} + \left( \frac{\gamma_m}{\gamma_m + \Gamma} \right) \frac{1}{C} \right) \]

\[ \Gamma << \gamma_m \quad \mathcal{I}_A = \mathcal{I}_F \]

Coupling

Noise from Exchange collisions

Spontaneous Emission noise

\[ C = \frac{g^2 n}{\kappa \gamma} >> 1 \]