Quo vadis Optica Quantorum?

Wanderin’ quantum optics theory (Warsaw, Saclay, Hannover, Barcelona)
**Theoretical Quantum Optics**

at the University of Hannover

at ICFO

**Cold atoms and cold gases:**
- Weakly interacting Bose and Fermi gases (solitons, vortices, phase fluctuations, atom optics, quantum engineering)
- Dipolar Bose and Fermi gases
- Collective cooling, CW atom laser, quantum master equation
- Strongly correlated systems in AMO physics

**Quantum Information:**
- Quantification and classification of entanglement
- Quantum cryptography and communications
- Implementations in quantum optics

**Matter in strong laser fields:**
- High harmonics generation, above threshold ionization, multielectron ionization
- Attophysics
- Analogies: Super-intense laser-atom physics and nonlinear atom optics
Quantum Optics is NOT a branch of theoretical physics. (heard frequently in Theory Institutes and in many other places)

Herr Lewenstein, Quantum Optics Fachverband will never get some of the DPG Prizes. For instance, the main theory prize, Max-Planck-Medaille, goes always to solid state and high energy people... (top figure of DPG, an experimentalist working in AMO physics, the day before Peter Zoller has become the recepient of the 2005 Max-Planck-Medaille)

Atomic physics and quantum optics beats condensed matter physics? (attributed to Wolfgang Ketterle)
1. Introduction (quotations and motivations):
   - I will present examples of the recent developments of Quantum Optics, and in particular of physics of ultra-cold gases, that occur at the interface between Quantum Optics, Atomic Physics, Quantum Information, Statistical Mechanics, Condensed Matter Physics, and even High Energy Physics, and touch the same frontiers and challenges of modern physics as other mentioned branches.

2. Ultracold gases and quantum frustrated antiferromagnets
   - Atomic gases in kagomé lattices
   - Quantum spin liquid crystals

3. Ultracold gases in non-abelian gauge fields
   - From Hofstatter butterfly to Hofstatter-Osterloh moth
   - Non-abelian interferometry
Atomic gases in Kagomé lattice: generating an „exotic“ spin liquid „in vivo“

L. Santos, M.A. Baranov, J.I. Cirac, H.-U. Everts, H. Fehrmann and M. Lewenstein,
Optical realization of kagomé lattice

Laser configuration

Optical potential
Why is it challenging?

Frustrated quantum antiferromagnets:

- Frustrated antiferromagnets lie at heart of modern condensed matter theory, and play essential role in theory of high $T_c$ superconductivity.
- According to C. Lhuillier and her colleagues there are 4 possible scenarios at low $T$:
  
  a) semiclassical Néel ordering;
  
  b) Valence Bond Crystal, with long range order of dimer coverings, no SU(2) symmetry breaking, short range spin, and long range dimer correlations;
  
  c) Resonating Valence Bond spin liquids of type I, with a unique ground state, no symmetry breaking, no long range order, and gapped “spinon” excitations;
  
  d) Resonating Valence Bond spin liquids of type II, with no symmetry breaking, no long range order, and an extraordinary density of low energy states.

- Heisenberg antiferromagnet in a trimerized kagomé lattice forms a paradigm example of the spin liquid of type II at low $T$. 
How to generate the Kagomé and *trimerized* Kagomé lattice?

One can change the geometry of the lattice, and thus the low energy quantum physics of the system in real time!!!

One may use the triple laser beam configuration (c) in three directions differing by $2\pi/3$ in the xy-plane, and phase shift one of the triplets!
Physics of atomic gases in trimerized kagomé lattice

- An atomic Bose gas in TKL exhibits a novel Mott insulator states with fractional filling 1/3 or 2/3. On the trimers the bosons (bosonic holes) form a W-state.

- A Fermi-Fermi mixture with half filling for both components represents a frustrated Heisenberg antiferromagnet with Resonating-Valence Bond ground state and quantum spin liquid of type II behavior dominated by continuous spectrum of singlet and triplet excitations.
Atomic Fermi gas in the trimerized kagomé lattice at the filling 2/3: quantum spin liquid-crystal

B. Damski, A. Honecker, H.-U. Everts, H. Fehrmann, L. Santos, and M. Lewenstein,

B. Damski, H.-U. Everts, H. Fehrmann, M.A. Baranov, L. Santos, and M. Lewenstein,
**Effective spin model**

- In the strongly trimerized Kagomé lattice at the filling 2/3 we have two fermions per trimer. One of them will occupy the zero momentum mode in the trimer, the other occupies left or right chirality mode. Effectively we deal with the spin $1/2$ at each trimer, and an effective spin model in the triangular lattice.

- Physics is governed by the Hamiltonian

$$H = J / 2 \sum_{i=1}^{N} \sum_{j=1}^{6} s_i(\phi_{i\rightarrow j}) s_j(\phi_{j\rightarrow i})$$

where $s_i(\phi) = \cos(\phi)s_{i,x} + \cos(\phi)s_{i,y}$, and the angles take values $2\pi/3$ or $-2\pi/3$, depending on the direction of the bond.
Classical and semiclassical theory ($s \to \infty$)

- Symmetries: $Z_6 = Z_3 Z_2$, where the first one combines the rotation of the lattice by $4\pi/3$ with the rotation of the spins by $2\pi/3$, while the second is the reflection of the spins in the x-y plane.

- For $J<0$, the “left” Néel-like state is the GS (see upper right). For $J>0$, spin wave analysis indicates that the ferromagnetic state along $j\pi/3$, $j=0,...,5$ is the GS (it competes with the “right” Néel-like state, and many other classical GS’s!).
Quantum theory (s=1/2)

- We diagonalize the Hamiltonian for 12-24 spins. For J<0, the “left” Néel-like state is the GS, and there exist a gap of order J.

- For J>0, the spin wave theory fails. The GS is the “right” Néel-like state. The Néel-like correlations survive at finite T=0.01J!
Quantum spin liquid-crystal

- The system exhibits for $J>0$ Néel-like correlations in the GS, and and at finite $T$. Is it just a normal antiferromagnet?

- For $J>0$, the system shows no gap (or a gap $<0.01J$) and extravagantly large number of low energy excitations. We term this novel behavior: quantum spin liquid-crystal!
Atomic lattice gases in non-abelian gauge fields: From Hofstadter butterfly to Osterloh cheese

K. Osterloh, M. Baig, L. Santos, P. Zoller, and M. Lewenstein,
Question: Can one mimic the effects of external magnetic field in an optical lattice?   YES!

1. What one needs is the „Peierls substitution“

\[ E(k) = -2J(\cos(k_x a) + \cos(k_y a)) \]

Replace \( k_x \rightarrow k_x - eA_x/\hbar c, \quad k_y \rightarrow k_y - eA_y/\hbar c, \)

2. With \( A=(0,Bx,0) \) we get the Schrödinger equation:

\[ E\Psi(m,n) = -J(\Psi(m+1,n) + \Psi(m-1,n) + e^{-i\alpha_m} \Psi(m,n+1) + e^{i\alpha_m} \Psi(m,n-1)) \]

where \( \alpha = a^2 B / 2\pi (\hbar c/e) \)

3. With \( \Psi(m,n) = e^{i\nu n} g(m) \) we obtain the Harper‘s equation

\[ \varepsilon g(m) = g(m+1) + g(m-1) + 2\cos(2\pi m\alpha - \nu) g(m) \]

Hofstadter butterfly

The scheme of Jaksch-Zoller

The scheme = combination of laser assisted tunneling, lattice tilting, employing of internal states

Magnetic flux $\alpha$
Question: Can one mimic the effects of external non-abelian U(N) gauge field in an optical lattice?

1. What one needs is the „U(N) Peierls substitution“
   \[ E(k) = -2J(\cos(k_x a) + \cos(k_y a)) \]
   Replace \( k_x \rightarrow k_x - e\hat{A}_x/\hbar c, k_y \rightarrow k_y - e\hat{A}_y/\hbar c \), where \( \hat{A} \) are matrix components of the gauge field vector potential. This can be easily done extending the schemes of Jaksch and others to atoms with N internal states (laser assisted tunneling!)

2. For instance, with \( \hat{A} = (M_1, M_2 x/a, 0) \), and \( M_2 = 2\pi \text{diag}(\alpha_1, \alpha_2, \ldots) \) we get the SU(N) generalization of the Harper‘s equation:
   \[ \varepsilon g(m) = \exp(iM_1)g(m+1) + \exp(-iM_1)g(m-1) + 2\cos(2\pi M_2 m - \nu)g(m) \]

3. For \( N=2 \) the spectrum plotted in the \( \alpha_1 - \alpha_2 \) plane for rational \( \alpha_1, \alpha_2 \) is mostly continuous, but the are complex patterns of gaps in it!
Hofstadter-Osterloh moth

Gaps in the energy spectrum for rational “magnetic fluxes”
Non-abelian (Aharonov-Bohm) interferometry

Phase=$\exp(2i\alpha L)$, L even
Phase=$\exp(2i\beta L)$, L odd

$U_y=\exp(i\alpha m)$

$\alpha=\text{diag}(\alpha_1, \alpha_2)$
$\beta=\text{diag}(\alpha_1, \alpha_2)$

Phase=depends on parity of L, but also y!!!

$U_y=\exp(i\alpha m)$

$\alpha=\text{diag}(\alpha_1, \alpha_2)$
$\beta=\text{diag}(\alpha_2, \alpha_1)$
Further challenges?

1. Can cold atoms be used to induce a non-abelian Aharonov-Bohm effect and non-abelian interferometry? **YES!**

2. Can cold atoms or ions be used to study lattice gauge theories?

   • One can generate „typical“ random configurations of gauge fields that are statistically relevant for various phases of the lattice gauge theories (area / perimeter laws for Wilson and t‘Hooft loops)
   • One can then study the behavior of matter (fermions, or bosons) in such fields

3. Can one generate a SU(N) monopoles (such as Polyakov monopoles) and study the dynamics of matter in such fields? ???, but see also the paper by Michael Fleischhauer et al. cond-mat/0503187
Hannover-Barcelona – Quantum Gases Theory

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CONCLUSIONS (The Tragedy of Hamlet, by Shakespeare):

• There are more thing in heaven and earth, Horatio, than are dreamt of in your philosophy.

Wow!!!