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Random Lasers

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Random Lasers

H. Cao et al., 1999:
Disordered ZnO-clusters and films

feedback by chaotic scattering of light
Theoretical Ideas

Lethokov: 1967
  – photon diffusion + linear gain

John, Wiersma & Lagendijk: 1995--
  – photon & atomic diffusion
  – non-linear coupling

Beenakker & coworkers: 1998--
  – random scattering approach
  – focus on linear regime below laser threshold
  – rate equations for strongly confined chaotic resonator

Soukoulis & coworkers: 2000--
  – numerical simulation of time-dependent Maxwell-equations coupled to active medium
issues to be discussed here:

- quantization for bad chaotic / disordered resonators

- increase of laser linewidth for bad resonators

- statistics for # of laser peaks

- statistics for # of emitted photons
Why modifications to laser theory?

- modes have simple spatial form
- approximation: eigenmodes of closed resonator
- modes have complex spatial distribution
- spectrally overlapping
field quantization for open resonators

\[ [a_\mu, a_\nu^\dagger] = \delta_{\mu\nu} \]

\[ [b_m(\omega), b_{m'}^{\dagger}(\omega')] = \delta_{mm'} \delta(\omega - \omega') \]

\[
H = \sum_\mu \hbar \omega_\mu a_\mu^\dagger a_\mu + \sum_m \int d\omega \hbar \omega b_m^{\dagger}(\omega)b_m(\omega)
\]

\[ + \hbar \sum_\mu \sum_m \int d\omega [W_{\mu m}(\omega) a_\mu^\dagger b_m(\omega) + V_{\mu m}(\omega) a_\mu b_m(\omega) + h.c.] \]

\( V_{\mu m}(\omega), W_{\mu m}(\omega) \): Integrals of mode functions along the openings

\( PRL \ 89, \ 083902 \ (2002), \ PRA \ 67, \ 013805 \ (2003) \)

Dynamics: Heisenberg picture

\[
\dot{a}_\mu = -i\omega_\mu a_\mu - \sum_\nu \kappa_{\mu\nu} a_\nu + F_\mu
\]

\[
\kappa = \pi WW^\dagger
\]

standard laser:

open laser:
Schrödinger picture, low temperature: \( k_B T < \hbar \omega \)

\[
\dot{\rho} = -\frac{i}{\hbar} \left[ \sum_{\mu} \hbar \omega_{\mu} a_{\mu}^\dagger a_{\mu}, \rho \right] + \sum_{\mu \nu} \kappa_{\mu \nu} \left\{ [a_{\mu}, \rho a_{\nu}^\dagger] + [a_{\nu} \rho, a_{\mu}^\dagger] \right\}
\]

G. Hackenbroich et al., PRA 013805 (2003): generalization of single-mode master equation to many overlapping modes
open laser

field-atom coupling (spatially random for chaotic modes)

non-linear coupled Heisenberg equations

\[ \dot{a}_\mu = \ldots \]  field modes

\[ \dot{S}_{-p} = \ldots \]  polarization of p-th atom

\[ \dot{S}_{zp} = \ldots \]  inversion of p-th atom

modes coupled not only via atoms but also through damping
increase of laser linewidth

\[ \delta \omega = K \delta \omega_{ST} \]

\[ K = \frac{\langle L \mid L \rangle \langle R \mid R \rangle}{|\langle L \mid R \rangle|^2} \geq 1 \]

regime of a single laser oscillation

Petermann factor

PRL 89, 083902 (2002):
multimode lasing and mode competition

modes of passive system overlap, fast atomic medium

\[ \dot{I}_k = -\kappa_k I_k + A_k I_k - \sum_{k'} B_{kk'} I_k I_{k'} \]

mode competition for atomic gain

\[ A_k \propto \int_{\text{cav}} dV \; L_k^*(\mathbf{r}) R_k(\mathbf{r}) \]

\[ B_{kk'} \propto V_{\text{cav}} \frac{\int_{\text{cav}} dV \; L_k^*(\mathbf{r}) R_k(\mathbf{r}) R_{k'}^*(\mathbf{r}) R_{k'}(\mathbf{r}) \int_{\text{cav}} dV \; R_k^*(\mathbf{r}) \int_{\text{cav}} dV \; R_{k'}(\mathbf{r})}{\int_{\text{cav}} dV \; L_k^*(\mathbf{r}) R_k(\mathbf{r}) \int_{\text{cav}} dV \; R_k^*(\mathbf{r}) \int_{\text{cav}} dV \; R_{k'}(\mathbf{r})} \]
mean number of lasing modes

\[ \langle N \rangle \]

\[ \langle N_{\text{no-comp}} \rangle \propto \varepsilon^{1/2} \]

\[ \langle N_{\text{las}} \rangle \propto \varepsilon^{1/3} \]

\[ \varepsilon = 1 \iff \text{linear gain} = \kappa_0 \]

- universal exponent: \( 1/3 \)
- non-linear mode competition relevant
summary

- theory of (weakly confined) random lasers
- generalization of standard laser theory
- characteristics:
  - mode coupling through damping
  - increase of laser linewidth
  - universal increase of # of lasing modes
  - increased intensity fluctuations in output *(not discussed here)*

thanks to: D. Savin, H. Cao
Random matrix-model

ensemble of non-Hermitian random matrices

\[ \mathcal{H} = H_{\text{GOE}} - i \pi W W^\dagger \]

Fyodorov & Sommers: 1997

\[ \kappa_0 = \frac{TM \Delta}{4\pi} \]

\[ P(\kappa) \sim \begin{cases} \kappa^{M/2-1}, & \kappa \ll \kappa_0 \\ 1/\kappa^2, & \kappa \gg \kappa_0 \end{cases} \]

\[ \langle B_{kk} \rangle = 1 + 2\delta_{kk} \]