Coherence and correlations in an atomic Mott insulator

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Outline

- Optical lattices and the superfluid to Mott insulator transition

- Phase coherence of a Mott insulator
  what does the interference pattern tell us about the nature of the ground state?

- Spatial correlations in expanding clouds
  two-particle correlations to probe phase-uncoherent samples
1D optical lattice

Potential: \[ V(x) = V_0 \sin^2(k_L x) \]

Natural scales:

- Recoil energy:
  \[ E_R = \frac{\hbar^2}{2m\lambda_L^2} \]
  \[ E_R/h \sim 3.2 \text{ kHz} \ (150 \text{ nK}) \]
  @ \( \lambda_L = 850 \text{ nm} \)

- Lattice spacing:
  \[ a_{\text{lat}} = \frac{\lambda_L}{2} = 425 \text{ nm} \]
3D Optical Lattices

- Three pairs of counter-propagating laser beams produce a simple cubic lattice
- Typically 20-60 sites occupied in each direction
- Mean atom number per site (filling factor) between 1 and 3
- Spontaneous emission rate ~ 1 Hz
Loading a BEC in the lattice

- Produce a $^{87}\text{Rb}$ Bose-Einstein condensate in a purely magnetic QUIC trap
- Expand the condensate to reduce its density (and avoid losses)
- Ramp up slowly lattice beams intensity
- Switch off the trap, expand and take an absorption image
Time of flight interference pattern

- Interference between all waves coherently emitted from each lattice site

\[-\frac{2\hbar k_L t}{m}\]

\[\tilde{n}(k) = |\tilde{w}(k)|^2 \sum_{i,j} e^{i k \cdot (r_i - r_j)} \alpha_i^{*} \alpha_j \]

Wannier envelope

Grating-like interference

Periodicity of the reciprocal lattice

Time of flight

20 ms
Reversible loss of coherence in deep lattices

Generalization to a general matter wave:

\[ \tilde{n}(k) = |\tilde{w}(k)|^2 \sum_{i,j} e^{ik \cdot (r_i - r_j)} \langle \hat{a}_i^\dagger \hat{a}_j \rangle \]

Correlation function determines the visibility

Phase coherence disappears with increasing lattice depth. This is reversible:

see also:
C. Orzel et al., Science 291, 2386 (2001)
Z. Hadzibabic et al., PRL 93, 180403 (2004)
Interactions matter: Bose-Hubbard model

Describes interacting Bose gas in a lattice, in the tight-binding limit

Competition between tunneling and on-site interactions:

\[ \mathcal{H} = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) \]

D. Jaksch et al., PRL 81, 3108 (1998)
Ground state in the zero tunneling limit

The system tries to form an atom distribution as regular as possible to minimize locally the interaction.

Mott insulator ground state

\[ |\Psi_0\rangle = \prod_i |n_0\rangle_i \]

- Integer number of atoms per site
- Zero fluctuations

Survives at finite temperature \(<< U\)
Intermediate regime

Superfluid ground state, \( J \ll U \)

\[
\langle n_i \rangle = 1 \\
\Delta n_i \sim \langle n_i \rangle
\]

Gapless excitations: compressible
Long-range phase coherence
Superfluid currents

Mott insulator ground state, \( U \gg J \)

\[
\langle n_i \rangle = 1 \\
\Delta n_i \sim 0
\]

Gapped excitations: incompressible
No off-diagonal long range order or superfluid currents
Phase coherence of a Mott insulator

Does a Mott insulator produce an interference pattern?

F. Gerbier et al., cond-mat/0503452, accepted in PRL.

Visibility of the interference pattern

\[ V = \frac{n_{\text{max}} - n_{\text{min}}}{n_{\text{max}} + n_{\text{min}}} \]

\[ d = 2v_{\text{tot}} \]

SF to MI transition
Excitations in the zero tunneling limit

Perfect Mott insulator ground state

\[ |\Psi_0\rangle = \prod_{i} |n_0\rangle_i \]

- Low energy excitations:
  - Particle/hole pairs couple to the ground state:
    \[ |\Psi_{\text{hole}}\rangle \propto \sum_i \hat{a}_i |\Psi_0\rangle \]
    \[ |\Psi_{\text{particle}}\rangle \propto \sum_i \hat{a}_i^\dagger |\Psi_0\rangle \]
  - \(n_0\): filling factor
    Here \(n_0=1\)

- Particle/hole pairs couples to the ground state:
  \[ |\Psi_{\text{ph}}\rangle \propto \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j |\Psi_0\rangle \]

Energy \(E_0+U\), separated from the ground state by an interaction gap \(U\)
Deviations from the perfect Mott Insulator

Ground state for $t \rightarrow 0$:
``perfect'' Mott insulator

$$|\Psi_0\rangle = \prod_i |n_0\rangle_i$$

Ground state for finite $t \ll U$:
treat the hopping term $H_{hop}$ in 1st order perturbation

$$|\Psi_1\rangle = - \sum_{n \neq g} \frac{H_{hop}}{E_g^{(0)} - E_n^{(0)}} |\Psi_0\rangle$$

$$= \frac{J}{U} + \frac{J}{U} + \ldots$$

*Coherent admixture of particle/holes at finite $t/U$*
Predictions for the visibility

Perfect MI

MI with particle/hole pairs

\[ V = 0 \]

\[ V \approx \frac{4}{3} \left( n_0 + 1 \right) \frac{zJ}{U} \]

Perturbation approach predicts a finite visibility, scaling as \((U/J)^{-1}\)
Comparison with experiments

Average slope measured to be $-0.97(7)$
A more careful theory

Many-body calculation for the homogeneous case

- 1st order calculation: admixture of particle/hole pairs to the MI bound to neighboring lattice sites

- Higher order in J/U: particle/holes excitations become mobile

Dispersion relation of the excitations is still characterized by an interaction gap.

One can obtain analytically the interference pattern (momentum distribution) for a given $n_0$.

More details in:
- D. van Oosten et al., PRA 63, 053601 (2001) and following papers
Shell structure of a trapped MI

Smooth "external" potential present on top of the lattice potential
(combination of magnetic trap + optical potential due to Gaussian profile)

Consequence: alternating MI/superfluid shells present at the same time

Figures courtesy of M. Niemeyer and H. Monien (Bonn)

D. Jaksch et al. PRL 81, 3108 (1998)
Comparison with experiments

Extends to trapped system using the Local Density Approximation

- Simplify shell structure:
  ignore superfluid rings

F. Gerbier *et al.*, in preparation

![Graph showing visibility V vs. lattice depth (ER)](image)
Kinks in the visibility curve: evidence for $n>1$ Mott shell formation?

**Experiment:**
- Kink #1 14.1 (8) Er
- Kink #2 16.6 (9) Er

**Theory:**
- $n=2$ Mott shell 14.7 Er
- $n=3$ Mott shell 15.9 Er

Reproduced in numerical calculations by the GSI Darmstadt group (R. Roth et al, unpublished)
Spatial correlations in expanding atom clouds

Hanbury Brown Twiss experiment

Seminal experiment by Hanbury-Brown and Twiss in 1952

Joint detection probability twice as large for superimposed detectors

Second order coherence function:

\[ g^{(2)}(\tau) = \frac{\langle I(t)I(t + \tau) \rangle}{\langle I(t) \rangle^2} \]

- \( g^{(2)} = 1 \) : uncorrelated particles
- \( g^{(2)} > 1 \) : bunching, typical for Bose statistics
Intensity interferometry via the Hanbury Brown and Twiss effect

- **Bunching as consequence of Bose statistics**

- (Quantum-statistical) Noise analysis as a **sensitive probe of the source properties**, with a wide range of applications:
  - Quantum optics
  - Nuclear and particle physics (angular correlations)
  - Condensed matter physics (electron antibunching, mesoscopics, …)

- **Emerging field in cold atom physics**
  - Masuda & Shimizu, PRL (1996);
  - Orsay (2005)
  - also pursued in optical cavity: Münich, Heidelberg, Zürich, Berkeley, …
Hidden information in expanding atom clouds

For a cloud deep in the Mott state (here $V_0=50\ E_R$), the interference pattern is unobservable.

Can we still extract information from such a picture?

The answer is yes, if we use noise analysis.
Correlated fluctuations in time of flight images

Bunching effect for relative distances equal to a reciprocal lattice vector

Correlation function (normalized)

\[ C(d) = \frac{\int d^2r \langle n_c(r + d/2)n_c(r - d/2) \rangle}{\int d^2r \langle n_c(r + d/2) \rangle \cdot \langle n_c(r - d/2) \rangle} \]
Hanbury Brown-Twiss Effect for Atoms (1)
Hanbury Brown-Twiss Effect for Atoms (2)

There's another way ...
Hanbury Brown-Twiss Effect for Atoms (3)

Cannot fundamentally distinguish between both paths...

\[ \phi = (k_1 \cdot r_{D_1} + k_2 \cdot r_{D_2}) - (k_2 \cdot r_{D_1} + k_1 \cdot r_{D_2}) = (k_1 - k_2) \cdot (r_{D_1} - r_{D_2}) \]

Relative phase accumulated when propagating from source to detector.

Two Particle Detection probability

\[ \pm e^{i\phi} \]
Hanbury Brown-Twiss Effect for Atoms (4)

Interference in Two-Particle Detection Probability

Detector 1

Detector 2

depends on source separation $a_{lat}$

$l = \frac{ht}{ma_{lat}}$
Multiple Wave Hanbury Brown-Twiss Effect

Interference in Two-Particle Detection Probability

Calculation for $N_s = 6$ sites

Detector 1 $\quad d$ $\quad a_{\text{lat}}$ $\quad$ Detector 2

$l = \frac{ht}{ma_{\text{lat}}}$
HBT theory predicts a factor of 2 enhancement of fluctuations, or \( C_{\text{max}} \approx 1 + 1 \).

In the experiment, the enhancement varies between \( 10^{-4} \) and \( 10^{-3} \)!

(Note the noise floor \( \sim 10^{-4} \)).

Atom density is in fact integrated over a column parallel to the probe.

In each bin, \( N_{\text{bin}} \gg 1 \) atoms are counted.

**Bin geometry:**

- \( w \) : cloud size
- \( \sigma \) : imaging resolution
How large are the correlations?

Coherence length:
also ideal peak width

\[ L_{\text{coh}} \sim \frac{\hbar t}{mN_s a_{\text{lat}}} = \frac{l}{N_s} \]

Great spatial resolution:
fringe spacing \( l \gg L_{\text{coh}} \gg \text{res.} \)
\[ C_{\text{max}} \approx 1 + 1 \]

Poor spatial resolution:
\( \text{res.} \gg \text{fringe spacing} \ l \gg L_{\text{coh}} \)
\[ C_{\text{max}} \approx 1 + \frac{1}{N_s^3} \]

Intermediate spatial resolution:
fringe spacing \( l \gg \text{res.} \gg L_{\text{coh}} \)
\[ C_{\text{max}} \approx 1 + \left( \frac{L_{\text{coh}}}{\text{res.}} \right)^3 \]

Imaging plane:
\( l \gg \sigma > L_{\text{coh}} \)

Probe direction:
\( w \gg l \)
Scaling of correlations

\[ C_{\text{max}} \approx 1 + \frac{1}{N} \left( \frac{l}{\sigma} \right)^2 \propto \frac{t_{\text{of}}^2}{N} \]

Comparison of the results to a more sophisticated model, taking shell structure into account:

- Scaling of the correlation amplitude with 1/N and \( t^2 \) approximately verified
- However correlation amplitude is too small by 40%
Applications to the detection of magnetic phases


Antiferromagnet (Bose/Fermi)

Spin waves (Bose/Fermi)

Charge density wave (predicted in Bosons/Fermions mixtures)
Conclusion and perspectives

- **Fundamental deviations from a perfect Mott state can be observed in the visibility**
  - Signature for particle/hole pairs
  - Evidence for $n>1$ shell formation?

  Implications for the fidelity of entanglement schemes in a lattice

- **Spatial correlations of density fluctuations in expanding clouds**
  - Signature of lattice ordering
  - Applications to the study of magnetic systems; also works for fermions
Other directions

• **Visibility in a 2D lattice**
  D. Gangardt *et al.*, *cond-mat* (2004): possible signature of correlations in each tube (Tonks-Girardeau)

• **Dynamical studies**

• **Resolve shell structure (microwave or rf spectroscopy)**

• **Detection of magnetic ordering**
Adiabatic or diabatic loading

How fast can we go to stay close to the ground state?
Loading a BEC in the lattice

• Produce a $^{87}$Rb Bose-Einstein condensate in a purely magnetic QUIC trap

• Expand the condensate to reduce its density (and avoid losses)

• Ramp up slowly lattice beams intensity

• Switch off the trap, expand and take an absorption image
Adiabatic loading in the lattice?

Smooth profile to ramp up the intensity of the lattice beams

Typically ramp time = 160 ms

Adiabaticity wrt the band structure: easy to fulfill (µs time scale)


Adiabaticity wrt many body dynamics?

S. Sklarz et al., PRA 66, 053620 (2002).
Influence of ramp time (SF regime)

Fix:
- Lattice depth $V_0 = 10 \, E_R$
- Hold time $t_{\text{hold}} = 300 \, \text{ms}$

Vary ramp time

Time constant $\sim 100 \, \text{ms}$
Much longer than microscopic time scales
Adiabaticity in the MI state

Compare calculated to measured visibility in the deep MI state

Breakdown around $V_0 \sim 25 \, E_R$, where
Tunneling time $\sim 200$ ms

- **Superfluid regime**: Time constant $\sim 100$ ms, much longer than tunneling time, trap frequencies, …
  $\Rightarrow$ Long-lived collective excitations involved

- **MI regime**: Breakdown of adiabaticity for lattice depth such that the tunneling time is comparable to ramp time
  $\Rightarrow$ Single particle redistribution