QUANTUM THEORY OF NOVEL PARAMETRIC DEVICES

P. D. Drummond, M. D. Reid, K. Dechoum, S. Chaturvedi, M. Olsen, K. Kheruntsyan, A. Bradley

Australian Centre for Quantum Atom Optics

June 17, 2005
ACQAO:

AUSTRALIAN CENTRE OF EXCELLENCE
FOR QUANTUM-ATOM OPTICS

• Nodes in Brisbane, Canberra, Melbourne

• Links with Paris, Dunedin, Hannover, Erlangen

• Postdoc, Ph.D., Diplom, Exchange projects..

• Worlds best squeezers, two BECs; He*, Fermi gas coming

www.ACQAO.org: Novel parametric devices
ACQAO in Queensland
NOVEL PARAMETRIC DEVICES

Twin-mode *limits to entanglement*

Planar *universal quantum critical fluctuations*

Nondegenerate planar *Mermin-Wagner theorem*

Coupled channel *integrated entanglement*

Cascade *triple correlations*

Parallel *triple entanglement*
INTRODUCTION: PARAMETRIC OSCILLATOR

- Down-converts $\omega_0 \rightarrow \omega_1 + \omega_2$, critical input field $E_c$
GENERIC PARAMETRIC HAMILTONIAN:

\[ \hat{H} = i\hbar \int d^n x \left[ \mathcal{E} \hat{\Psi}_0^\dagger(x) + \chi \hat{\Psi}_1^\dagger(x) \hat{\Psi}_2^\dagger(x) \hat{\Psi}_0(x) \right] - h.c \]

- \( \hat{\Psi}_0 \), \( \hat{\Psi}_1 \) and \( \hat{\Psi}_2 \) are modes at \( \omega_0 \), \( \omega_1 \) and \( \omega_2 \) (\( \omega_0 = \omega_1 + \omega_2 \))

- \( \mathcal{E} \) is a coherent pump laser at \( \omega_0 \); \( n \) is transverse dimension

- \( \chi \) is the nonlinear second order polarizability constant
**MASTER EQUATION:**

Includes coupling to external reservoirs (output couplers)

\[
\frac{d\hat{\rho}}{dt} = \frac{1}{i\hbar} [\hat{H}, \hat{\rho}] + \sum_{i=0}^{2} \gamma_i \int d^n x \left[ 2\hat{\Psi}_i \hat{\rho} \hat{\Psi}_i^\dagger - \hat{\Psi}_i^\dagger \hat{\Psi}_i \hat{\rho} - \hat{\rho} \hat{\Psi}_i^\dagger \hat{\Psi}_i \right]
\]

- $\hat{\rho}$ is the density matrix
- $\gamma_i$ is the damping of the $i$–th mode (assume $\gamma_1 = \gamma_2$); $\gamma_r = \gamma_0 / \gamma_1$
OBSERVABLES and EPR:

• Output fields: $\hat{\Phi}_j(t) = \sqrt{2\gamma_j} \hat{a}_j(t) - \hat{\Phi}^{\text{in}}_j(t)$

• Measured quadratures: $\hat{X}^\theta_j = e^{-i\theta}\hat{\Phi}_j + e^{i\theta}\hat{\Phi}^\dagger_j$

• Heisenberg Uncertainty Principle: $\Delta X_j \Delta Y_j \geq 1$

• Original ‘EPR’ criterion: $X_1 = X_2$ and $Y_1 = -Y_2$

• EITHER local realism fails OR QM is incomplete

✗ Problem: can’t occur physically - infinite energy!
EPR Paradox: is it physically realizable?
‘INFERRED’ HEISENBERG CRITERION

✔ EPR paradox with finite energy (Reid-Drummond):

\[ \Delta_{inf}X_1\Delta_{inf}Y_1 < 1 \]

- Combined variance: \( V^{\pi/2}(0) = \frac{1}{4} \left\langle \left[ \hat{Y}_1 + \hat{Y}_2 \right]^2 + \left[ \hat{X}_1 - \hat{X}_2 \right]^2 \right\rangle \)

- Inferred HUP \( \rightarrow \) \( 2V^0 V^{\pi/2} < V^0 + V^{\pi/2} \rightarrow \) EPR paradox

- Entanglement criterion: \( V^\theta < 1 \rightarrow \) demonstrates inseparability
POSITIVE-P REPRESENTATION:

\[
\frac{d\alpha_0}{d\tau} = \mathcal{E} - \tilde{\gamma}_0 \alpha_0 - g\alpha_1 \alpha_2
\]

\[
\frac{d\alpha_i}{d\tau} = -\tilde{\gamma}_i \alpha_i + g\alpha_{3-i} \alpha_0 + \sqrt{g\alpha_0} \zeta_i(\tau)
\]

• NOISE TERM: \( \langle \zeta_i(\tau, x) \zeta_j(\tau', x') \rangle = \delta^n(x - x') \delta(\tau - \tau') \delta_{i,3-j} \)

• LINEAR TERM: \( \tilde{\gamma}_j = \left[ \gamma_j - i\Delta_j - i\frac{\nu^2}{2\omega_j} \nabla^2 \right] / \gamma_1 \) \( (\tau = \gamma_1 t) \)

• NONLINEAR TERM: \( g^2 = \chi / (2\gamma_0 \gamma_1) \)
STOCHASTIC DIAGRAM SOLUTION

Expand as a power series in $g$: small parameter expansion

$$z_j(\tau) = \sum_{n=0}^{\infty} g^{n/2} z_j^{(n)}(\tau).$$

$$X^{(0)}(t) = \rightarrow$$

$$X^{(1)}(t) = \rightarrow + \rightarrow + \rightarrow$$

Fig. 1
1. TWIN-MODE PARAMP

Predicted external squeezing variance BEYOND GAUSSIAN:

\[ V^{\pi/2} = 1 - \frac{4\mu}{(1+\mu)^2} + \frac{4g^2\mu}{(1+\mu)^4} \left[ 1 + \frac{2\mu^2\gamma_r(2+\gamma_r)}{(1-\mu)((1+\gamma_r)^2 - \mu^2)} \right]. \]

Also get NONCLASSICAL TRIPLE CORRELATIONS

\[ \langle \hat{X}\hat{Y}^{\dagger}\hat{Y}_0 \rangle = \propto \frac{g\mu^2}{(1-\mu)^2}. \]
PHASE INFORMATION IS CONSERVED!

Take high-Q pump: $\gamma_r \rightarrow 0$. What is the smallest variance?

$$V \simeq g \sqrt{\gamma_r/2} \simeq \frac{1}{\sqrt{N_c}}.$$  

PHYSICS: $N_c =$ photons used per coherence time

- INPUT PHASE UNCERTAINTY: $\Delta\theta_{in} = 1/\sqrt{N_t}$

✓ OUTPUT PHASE UNCERTAINTY: $\Delta\theta_{out} = \Delta Y / \Delta X = 1/V$
OPTIMUM ENTANGLEMENT, $g^2 = 0.001, \gamma_r = 0.01$
Summary

- EPR entanglement optimised below critical point, *high-Q* pump

- 30dB squeezing for $g^2 = 0.001, \gamma_r = 0.01$

- INFODYNAMIC Limit: phase information is conserved

- Non-classical triple correlations

- Requires small, efficient devices near threshold

- Dechoum et al PHYS REV A 2004
2. PLANAR DEGENERATE PARAMP

Studied by Lugiato et al using linearization.

- What happens if we go near the critical point?

- Need to include nonlinear terms!

\[
\frac{\partial X}{\partial \tau} = - \left[ \gamma_x + X^2 + g_c Y^2 \right] X - \left[ \gamma_{xy} + \nabla^2 \right] Y + \xi_x
\]

\[
g_c \frac{\partial Y}{\partial \tau} = - \left[ \gamma_y + g_c X^2 + g_c^2 Y^2 \right] Y - \left[ \gamma_{yx} - \nabla^2 \right] X + \xi_y.
\]
Critical fluctuations for *lossy* pump: $\gamma_r \to \infty$

\[
\frac{\partial X}{\partial \tau} = -\gamma_x X - X^3 - \frac{\gamma_{xy}}{2} \nabla^2 X - \frac{1}{2} \nabla^4 X + \xi_x .
\]

Exact solution for the probability density $P[X]$, 

\[
P \propto \exp \left[ \frac{-1}{4} \int d^2 \vec{r} \left( 2\gamma_x X^2 + X^4 - \gamma_{xy} [\nabla X]^2 + [\nabla^2 X]^2 \right) \right]
\]
Fluctuations

\[ \langle |X(k)|^2 \rangle \]
Entanglement spectrum

\[ V(\Omega) = 1 - \frac{1 - g_c(\langle X^2 \rangle + \gamma_x)}{(g_c\Omega/2)^2 + (1 + g_c(\langle X^2 \rangle - \gamma_x)/2)^2}. \]

- Quantum features NOT found in conventional Lifshitz theory
- Entanglement is REDUCED due to critical fluctuations
- This is NOT predicted in linearized treatments!
Entanglement: full theory vs linearized

![Graph showing V(Ω) vs gc Ω]
Summary

- IDENTICAL to the free-energy of magnetic Lifshitz points
- Driving field $\rightarrow$ Inverse Temperature (order-disorder transition)
- Entanglement reduced by critical fluctuations
- Drummond & Dechoum, P.R.L. (to appear, 2005)
3. PLANAR NON-DEGENERATE PARAMP

- What happens with TYPE II downconversion?
- Two possible polarizations!
- Lifshitz point with two-dimensional order parameter
- Mermin-Wagner theorem --> phase-waves destroy order
- NO PHASE TRANSITION
Summary: PHASE TRANSITIONS?

When do we have a non-equilibrium quantum phase transition?

<table>
<thead>
<tr>
<th>TRANSVERSE DIMENSION</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEGENERATE</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>NON-DEGENERATE</td>
<td>NO</td>
<td>NO</td>
</tr>
</tbody>
</table>
4. INTEGRATED EVANESCENT-WAVE PARAMP
Effective Hamiltonian:

\[ \mathcal{H}_{\text{eff}} = \mathcal{H}_{\text{int}} + \mathcal{H}_{\text{couple}} + \mathcal{H}_{\text{pump}} + \mathcal{H}_{\text{res}}, \]

Nonlinear interaction:

\[ \mathcal{H}_{\text{int}} = i\hbar \kappa \left[ \hat{a}_{1}^{\dagger} \hat{b}_{1}^{2} - \hat{a}_{1}^{2} \hat{b}_{1}^{\dagger} + \hat{a}_{2}^{\dagger} \hat{b}_{2}^{2} - \hat{a}_{2}^{2} \hat{b}_{2}^{\dagger} \right]. \]

Evanescent wave coupling:

\[ \mathcal{H}_{\text{couple}} = \hbar J_{a} \left[ \hat{a}_{1}^{\dagger} \hat{a}_{2} + \hat{a}_{1}^{\dagger} \hat{a}_{2}^{\dagger n} \right] + \hbar J_{b} \left[ \hat{b}_{1}^{\dagger} \hat{b}_{2} + \hat{b}_{1}^{\dagger} \hat{b}_{2}^{\dagger} \right], \]
Results: entanglement vs detuning

\[
S_{\text{out}}(X) + S_{\text{out}}(Y)
\]

\(\omega\) (units of \(\gamma\))
5. CASCADE PARAMP

What about using two paramps in series inside a cavity?
\begin{align*}
\dot{z}_0 &= \gamma_r (\mu - z_0) - z_1 z_2, \\
\dot{z}_1 &= -z_1 + z_0 z_2^+ + g \sqrt{z_0} \zeta_1(\tau), \\
\dot{z}_2 &= -z_2 + z_0 z_1^+ - z_3 z_4 + g \sqrt{z_0} \zeta_2(\tau), \\
\dot{z}_3 &= -z_3 + z_2 z_4^+ + g \sqrt{z_2} \zeta_3(\tau), \\
\dot{z}_4 &= -z_4 + z_2 z_3^+ + g \sqrt{z_2} \zeta_4(\tau).
\end{align*}
Phase Diagram: two thresholds

\[ \varepsilon^2 = \frac{|E_0|^2}{|E_{\text{thr},1}|^2} \]
Summary

- Two successive threshold transitions
- Strong quantum triple correlations below threshold
- Truncated Wigner theory has no correlations of the same order!
6. Parallel Paramp

What about using three paramaps each with a pair of common modes?

✔ Theory: use +P representation

✔ Get strong, multi-partite entangled modes

✔ Experiment: Pfister in Virginia has NSF grants for experiment.
EXPERIMENTS

✔ Below-threshold experiments: Kimble (Caltech), Bachor (ANU)

✔ Critical, cascade, parallel paramps: Pfister (Virginia)

✔ Transverse dynamics: Fabre (Paris)

• NEED EXCELLENT LASER STABILITY!
ACQAO Entangler Experiment

www.ACQAO.org: Novel parametric devices
Nonlinear parametric devices can:

- **generate** large amounts of entanglement,
- **show** Lifshitz phase transitions
- **explore** critical universality classes,
- **become** integrated
- **develop** multiport correlated states
- **demonstrate** multipartite entanglement