Problem 1: Number of gluons

Number of quanta in harmonic oscillator: $N = E/\hbar\omega$

$$N(x,p) = \frac{p^2/m + m\omega^2 x^2}{2\hbar\omega}$$

Wigner function of the ground state is

$$W(x,p) = \frac{1}{\pi\hbar}\exp(-2N(x,p))$$

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Number of photons: $N = \sum_\lambda \int d^3k \, c_\lambda^\dagger(\boldsymbol{k}) c_\lambda(\boldsymbol{k})$

$$N[\mathbf{A}, \mathbf{D}] = \frac{1}{4\pi^2\hbar} \int d^3r \int d^3r' \times \left(\sqrt{\frac{\epsilon}{\mu}} \mathbf{B}(\mathbf{r}) \frac{1}{|\mathbf{r}-\mathbf{r}'|^2} \cdot \mathbf{B}(\mathbf{r}') + \sqrt{\frac{\mu}{\epsilon}} \mathbf{D}(\mathbf{r}) \frac{1}{|\mathbf{r}-\mathbf{r}'|^2} \cdot \mathbf{D}(\mathbf{r}') \right)$$

Wigner functional of the electromagnetic vacuum

$$W[\mathbf{A}, \mathbf{D}] = \exp\left(- \frac{1}{2\pi^2\hbar} \int d^3r \int d^3r' \times \left(\sqrt{\frac{\epsilon}{\mu}} \mathbf{B}(\mathbf{r}) \frac{1}{|\mathbf{r}-\mathbf{r}'|^2} \cdot \mathbf{B}(\mathbf{r}') + \sqrt{\frac{\mu}{\epsilon}} \mathbf{D}(\mathbf{r}) \frac{1}{|\mathbf{r}-\mathbf{r}'|^2} \cdot \mathbf{D}(\mathbf{r}') \right) \right)$$

The Wigner functional of the electromagnetic field
Optics Communications **179**, 237 (2000)

Find the corresponding expression for the gluon field

Problem 2: Entropic uncertainty relation

Shannon measure of information

$$H = -\sum_i p_i \ln p_i$$

Information about position and momentum

$$H^{(x)} = -\sum p_l \ln p_l \quad H^{(p)} = -\sum \tilde{p}_k \ln \tilde{p}_k$$

$$p_l = \int_{x_l}^{x_l+\delta x} dx \, |\psi(x)|^2 \quad \tilde{p}_k = \int_{p_k}^{p_k+\delta p} dp \, |\tilde{\psi}(p)|^2$$

Localization is position: $H^{(x)} = 0$
Localization is momentum: $H^{(p)} = 0$

Entropic uncertainty relation

$$H^{(x)} + H^{(p)} > 1 - \ln 2 - \ln\gamma + O(\gamma) \quad \gamma = \frac{\delta x \delta p}{h}$$

Formulation of the uncertainty relations in terms of Rényi entropies
Physical Review A **74**, 052101 (2006)

$$-\int_{-\infty}^{\infty} dx\, |\psi(x)|^2 \ln|\psi(x)|^2 - \int_{-\infty}^{\infty} dp\, |\tilde{\psi}(p)|^2 \ln|\tilde{\psi}(p)|^2 \geq \ln(e\pi)$$

Find function $B(\gamma)$ that saturates the inequality

$$H^{(x)} + H^{(p)} \geq B(\gamma)$$

Relevant papers available on my homepage at www.cft.edu.pl