Particle beams guided by electromagnetic vortices

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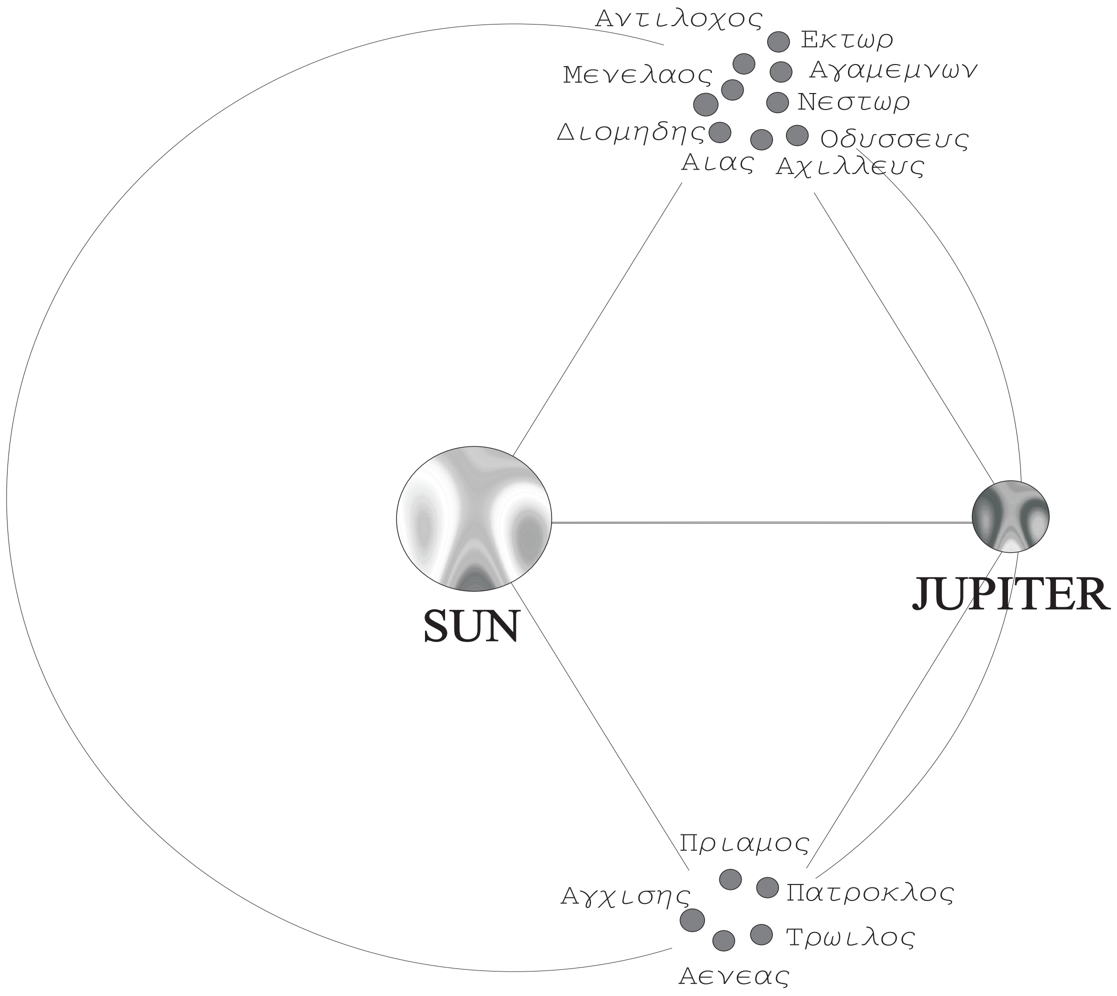
Calendar of discoveries

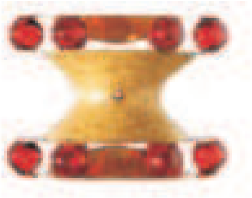
Giuseppe Lodovico Lagrangia 1772
Edward E. Barnard (Yerkes Observatory) 1904
Max Wolf (Heidelberg Observatory)1906

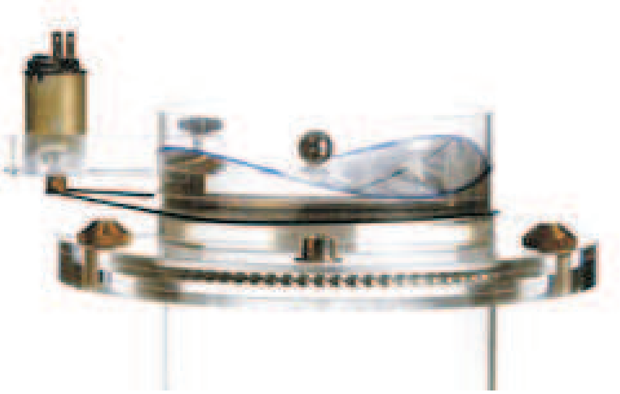
233 years ago prediction of the points of equilibrium in 3-body systems
101 years ago the first observation of a Trojan asteroid
99 years ago discovery of the largest Trojan asteroids



W. Paul and H. Steinwedel 1953

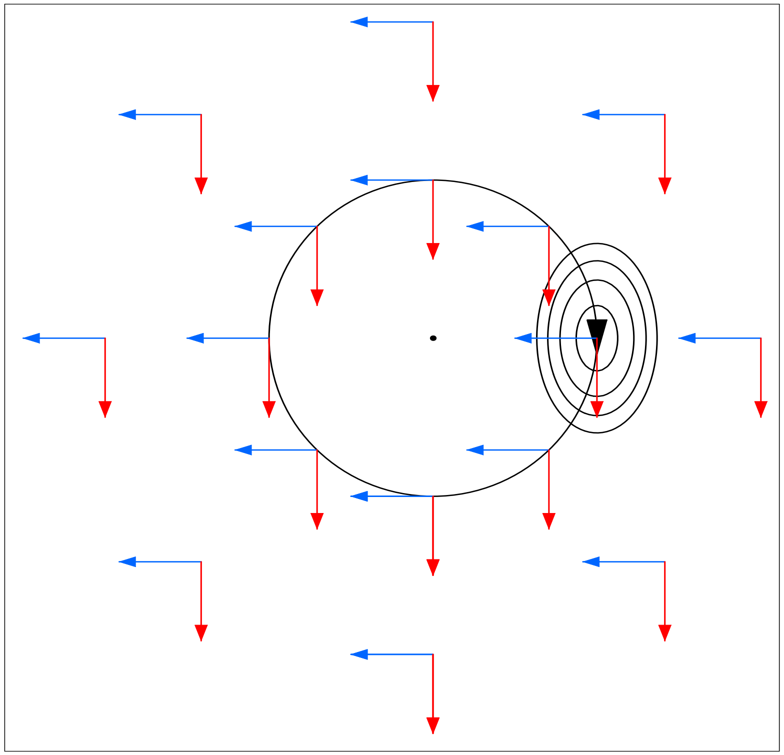
52 years of the Paul trap — Nobel Prize for Paul 1989





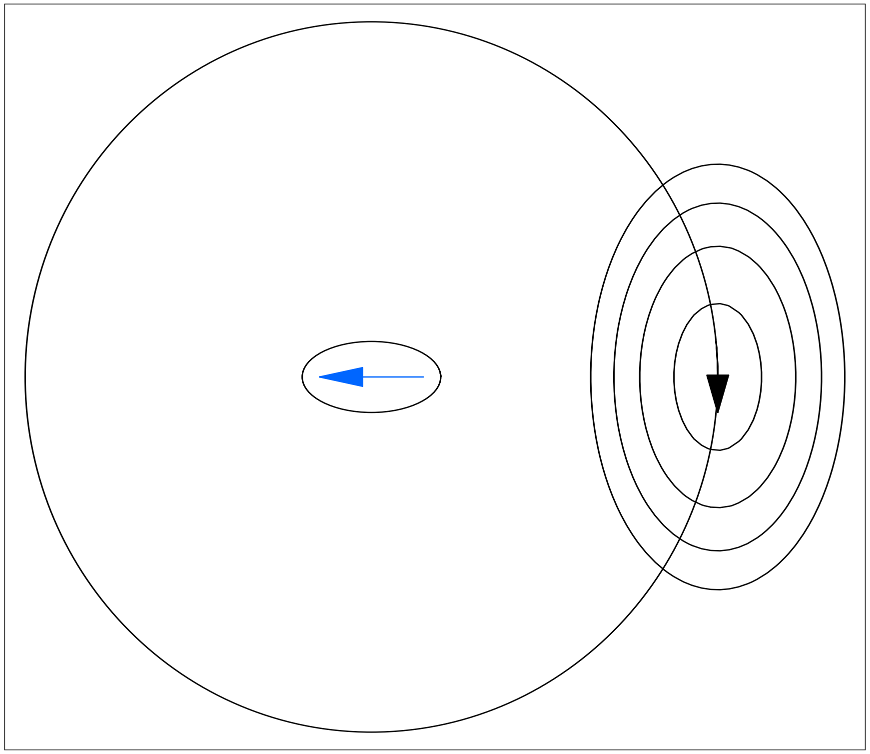
Real electromagnetic trap (top) and
the mechanical model made for the Paul's Nobel lecture
Theory by E. T. Whittaker in Analytical Dynamics 1904

11 years of the theoretical prediction of Trojan wave packets for Rydberg electrons in a circularly polarized wave



I. Bialynicki-Birula, M. Kalinski and J. H. Eberly 1994

Nonspreading wave packets of Rydberg electrons in rotating molecules with electric dipole moments



I. Bialynicki-Birula and Z. Bialynicka-Birula 1996

Trojan states finally experimentally observed!

Nondispersing wave packets

H. Maeda and T. F. Gallager, PRL 92, 133004 (2004)

Trojan states are still alive!

Nondispersive two-electron wave packets in helium atoms

M. Kalinski, L. Hansen, and D. Farelly
PRL September 2005

2-D oscillator in a rotating frame of reference

Equations of motion

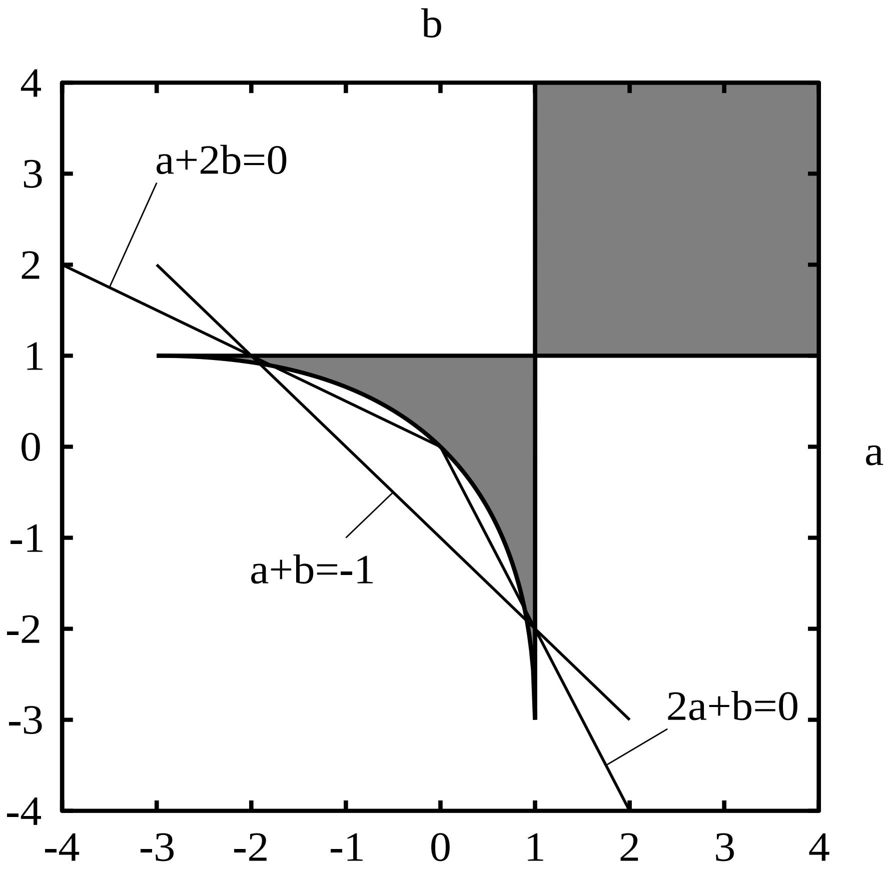
$$dx/dt = p_x/m + wy \qquad dp_x/dt = -ma\,x + wp_y$$
$$dy/dt = p_y/m - wx \qquad dp_y/dt = -mb\,y - wp_x$$

Two eigenfrequencies

$$\Omega_\pm = \sqrt{2w^2 + a + b \pm \sqrt{(a-b)^2 + 8(a+b)w^2}}\Big/\sqrt{2}$$

Diagonal form of the Hamiltonian

$$H = \Omega_+ a_+^* a_+ - \Omega_- a_-^* a_-$$


b
a
a+2b=0
a+b=-1
2a+b=0
4
3
2
1
0
-1
-2
-3
-4
-4
-3
-2
-1
0
1
2
3
4

What's new?

I will show that Trojan states of charged particle may exist
even without the central body such as the Sun or the nucleus
They are found in the presence of an electromagnetic wave
endowed with a vortex line

What is an electromagnetic vortex line?
There are several answers to this question
Today I will use the definition that is the closest
to the one used in quantum mechanics

$$\vec{v} = \frac{\vec{j}}{\rho}, \quad \vec{v} = \frac{\hbar}{2mi}\frac{\psi^* \overset{\leftrightarrow}{\nabla} \psi}{\psi^* \psi}$$

The signature of a vortex line is a nonvanishing vorticity

$$\oint d\vec{l} \cdot \vec{v} \neq 0$$

Electromagnetic vortex lines

Riemann-Silberstein vector

$$\vec{F} = \sqrt{\frac{\epsilon_0}{2}}(\vec{E} + ic\vec{B})$$

$$\begin{aligned} i\partial_t \vec{F} &= c\nabla \times \vec{F} \\ \nabla \cdot \vec{F} &= 0 \end{aligned}$$

In electromagnetism $\vec{F}$ plays the role of ψ

$$\vec{v} = \frac{1}{2i}\frac{F_i^* \overset{\leftrightarrow}{\nabla} F_i}{\vec{F}^* \cdot \vec{F}}$$

Simplest EM field with a vortex line

$$\vec{F} = \phi\,\vec{F}_0$$

Single scalar function ϕ controls vortex lines
Class of solutions of Maxwell equations

$$\vec{F}(\vec{r},t) = \phi(x+iy, \omega t - kz)(\hat{x}+i\hat{y})$$

Simplest case: Vortex line of a unit strength

$$\vec{F}(\vec{r},t) = (x+iy)(\hat{x}+i\hat{y})\exp(-i\omega t + ikz)$$

$$\begin{aligned} \vec{E}(\vec{r},t)/c &= f(\rho,\varphi,z,t)\,\hat{x} + g(\rho,\varphi,z,t)\,\hat{y} \\ \vec{B}(\vec{r},t) &= -g(\rho,\varphi,z,t)\,\hat{x} + f(\rho,\varphi,z,t)\,\hat{y} \end{aligned}$$

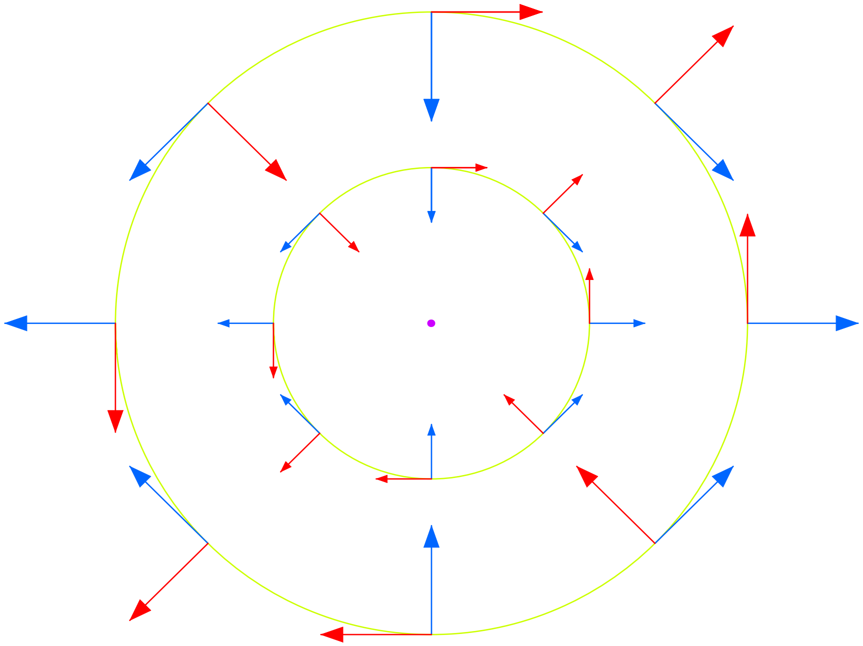
$$\begin{aligned} f(\rho,\varphi,z,t) &= B_0 k\rho\cos(\omega(t-z/c)-\varphi) \\ g(\rho,\varphi,z,t) &= B_0 k\rho\sin(\omega(t-z/c)-\varphi) \end{aligned}$$

Anatomy of my vortex solution

For fixed values of polar coordinates ρ and φ it looks
like a circularly polarized wave but its amplitude
increases with the distance from the z-axis
and its phase (defined as the direction of the EB pair)
increases as we move clockwise around the z-axis

As we move around the z axis
the vectors $\vec{E}$ and $\vec{B}$ rotate by 2π
This is the signature of a vortex line with unit vorticity

The field vectors also rotate in time and in the z direction
Rotation of the field vectors in time and in the z variable
can be frozen by a transformation to the comoving frame
In this noninertial frame the fields are time independent
but there are additional (inertial) forces acting on the particle



Classical beam dynamics

$$m\,\ddot{\xi}^{\mu}(\tau) = e\,f^{\mu\nu}(\xi(\tau))\dot{\xi}_{\nu}(\tau)$$

$$\frac{d\vec{p}(t)}{dt} = e(\vec{E} + \vec{v} \times \vec{B})$$

$$\begin{aligned}
\ddot{\xi}(\tau) &= \omega_c\,\omega\,f(\xi,\eta,\zeta,\theta)\left(\dot{\theta}(\tau) - \dot{\zeta}(\tau)/c\right) + \Omega_c\dot{\eta}(\tau) \\
\ddot{\eta}(\tau) &= \omega_c\,\omega\,g(\xi,\eta,\zeta,\theta)\left(\dot{\theta}(\tau) - \dot{\zeta}(\tau)/c\right) - \Omega_c\dot{\xi}(\tau) \\
\ddot{\zeta}(\tau) &= \frac{\omega_c\,\omega}{c}\left(\dot{\xi}(\tau)f(\xi,\eta,\zeta,\theta) + \dot{\eta}(\tau)g(\xi,\eta,\zeta,\theta)\right) \\
c\,\ddot{\theta}(\tau) &= \frac{\omega_c\,\omega}{c}\left(\dot{\xi}(\tau)f(\xi,\eta,\zeta,\theta) + \dot{\eta}(\tau)g(\xi,\eta,\zeta,\theta)\right)
\end{aligned}$$

Wave field amplitude ω_c and constant magnetic field Ω_c
measured by the corresponding cyclotron frequencies

Transverse dynamics

2-D oscillator Hamiltonian in the rotating frame

$$H_{\rm osc} = \frac{p_x^2 + p_y^2}{2m} + m\frac{a\,x^2 + b\,y^2}{2} - w(xp_y - yp_x)$$

Transverse part of the Hamiltonian in the rotating frame in the presence of a beam and a constant magnetic field

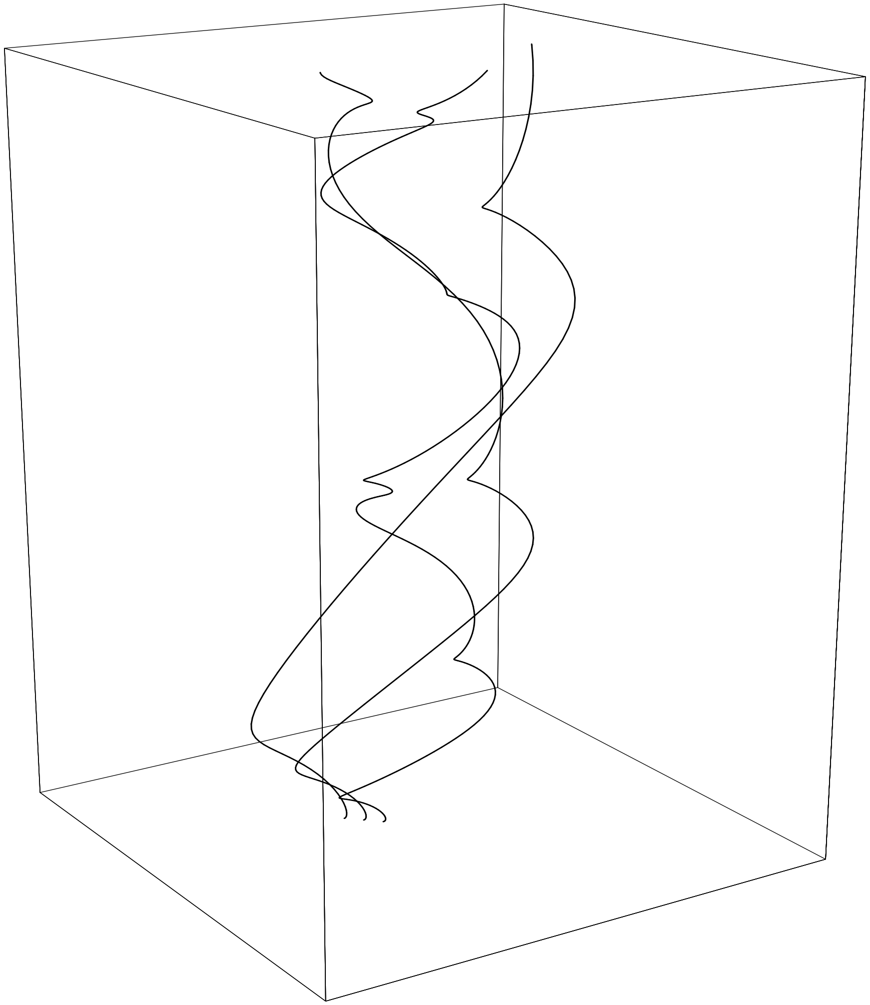
$$a = \frac{\Omega_c^2}{4} - \Omega\omega_c, \quad b = \frac{\Omega_c^2}{4} + \Omega\omega_c, \quad w = \frac{\Omega+\Omega_c}{2}$$

$$\omega_c = \frac{eB_0}{m}, \quad \Omega_c = \frac{eB_1}{m}, \quad \Omega = \omega\frac{E_p - cp_z}{mc^2}$$

$$H = \Omega_+ a_+^* a_+ - \Omega_- a_-^* a_-$$

$$\Omega_\pm = \frac{1}{2}\sqrt{(\Omega+\Omega_c)^2 + \Omega_c^2 \pm 2\sqrt{4(\omega_c\Omega)^2 + \Omega_c^2(\Omega+\Omega_c)^2}}$$

Classical trajectories



Quantum beam dynamics (Klein-Gordon)

$$\frac{4}{c^2}\frac{\partial^2}{\partial t_+ \partial t_-}\Psi = \left(\Delta_\perp - \frac{e^2}{\hbar^2}\vec{A}^2 - 2i\frac{e}{\hbar}\vec{A}\cdot\nabla - \frac{m^2c^2}{\hbar^2}\right)\Psi$$

$$t_\pm = t \pm z/c \qquad \vec{A}(\vec{r},t) = k^{-1}\vec{B}(\vec{r},t) - \frac{1}{2}\vec{r}\times\vec{B}_0$$

Solution is simple in the comoving frame

$$\Psi = e^{-ic^2(Mt_+ + m^2 t_-/M)/2\hbar}\, e^{-i\omega t_-(x\partial_y - y\partial_x)/2}\, e^{-ifxy}\,\Phi$$

$$i\hbar\frac{\partial}{\partial t_-}\Phi = \left(-\frac{\hbar^2}{2M}\Delta_\perp + M\frac{ax^2 + by^2}{2} + i\hbar w(x\partial_y - y\partial_x)\right)\Phi$$

Quadratic Hamiltonians possess solutions of the harmonic oscillator type

$$|n, m\rangle = (\hat{a}_+^\dagger)^n(\hat{a}_-^\dagger)^m|0\rangle$$

Quantum beam dynamics (Dirac)

Reduction of the Dirac equation to equations for two-component spinors

$$\Psi = \tfrac{1+\alpha_z}{2}\Psi_+ + \tfrac{1-\alpha_z}{2}\Psi_-$$

$$2i\hbar\partial_+\Psi_+ = c\left(mc\sigma_z - \boldsymbol{\sigma}_\perp\cdot(i\hbar\nabla + e\boldsymbol{A})\right)\Psi_-,$$

$$2i\hbar\partial_-\Psi_- = c\left(mc\sigma_z - \boldsymbol{\sigma}_\perp\cdot(i\hbar\nabla + e\boldsymbol{A})\right)\Psi_+,$$

$$\Psi_\pm = e^{-ic^2(Mt_+ + m^2t_-/M)/2\hbar}e^{-\omega t_-(x\partial_y - y\partial_x + i\sigma_z)/2}e^{-ifxy}\Phi_\pm$$

$$i\hbar\frac{\partial}{\partial t_-}\Phi_- = \left(-\frac{\hbar^2}{2M}\Delta_\perp + M\frac{ax^2+by^2}{2} + i\hbar w(x\partial_y - y\partial_x) - \frac{\hbar w}{2}\sigma_z\right)\Phi_-$$

Again we obtain a Schrödinger-type equation for an exactly soluble problem
Anisotropic harmonic oscillator in a rotating frame of reference

Conclusions

Beams of electromagnetic radiation with a vortex line trap charged particles

Trapping is caused by the “Trojan mechanism”:
Coriolis force in the frame rotating with the field vectors or a stabilization of orbits due to a rapid change of direction of the force in the laboratory frame

Trapping takes place only in a restricted range of parameters
The field strength of the EM wave cannot be too large