Can homodyne principle help to understand quantum measurement

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Credo

Fundamental problems of quantum theory (measurement, interpretation, scope, etc.) cannot be resolved within oversimplified models

Two-dimensional Hilbert space wouldn't do!

The homodyne method uses a local oscillator that is tuned to the carrier frequency of the desired station and mixed with the incoming signal to boost the power of that frequency before being filtered.

The key terms are: “**carrier frequency**” and “**local oscillator**”

What is the role of the **carrier wave**?
Without the carrier wave radio transmissions would suffer from very serious problems.
Consider the electromagnetic broadcast of an acoustic signal (speech or music).

There is a great disparity between the **propagation velocity** of sound waves and electromagnetic waves.

This disparity causes a six orders of magnitude difference between the wave length of the acoustic and electromagnetic signals.

The standard 440 Hz pitch produces the sound with its wavelength in the air of about **0.7m** while one wavelength of an electromagnetic wave with this frequency would wrap the Earth at the equator **sixteen times**. The broadcasting and reception of such long waves would be impossible.

Carrier waves having different frequencies allow us to separate signals from different sources. Without this separation we would have a **Tower of Babel effect** and a selective reception would be out of question. We would just hear white noise, like during a coffee break at a conference.

What is the connection between radio transmissions and quantum mechanics?

In both cases we have a **carrier wave** and its modulation.

In their everyday work, most physicists are not even aware that the standard **Schrödinger equation** is an approximate equation—it is only an equation for the **envelope**—the carrier wave has been eliminated and it is completely hidden from view.

In Encyclopedia Britannica we read that the Schrödinger equation is

"**the fundamental equation** of the science of submicroscopic phenomena known as quantum mechanics. The equation, developed (1926) by the Austrian physicist Erwin Schrödinger, has the same **central importance** to quantum mechanics as Newtons laws of motion have for the large-scale phenomena of classical mechanics".

I do not question the **central importance**e of the Schrödinger equation, but I only deny its **fundamental nature**.

The fundamental equation is in my view
the Dirac equation
which describes both
the carrier wave and its modulation.

I have to admit that I do not have a profound explanation of the role of the **carrier wave** but the unobservable carrier frequency is present.

There are still more questions than answers, but I believe that the questions are worth asking and the search for answers may lead to a better **understanding of the quantum world**.

I shall write the **Dirac equation** as two coupled equations for two **real** four-component Majorana spinors χ_1 and χ_2

I have chosen this representation to separate the issue of the role of complex numbers in quantum theory from the problem of carrier wave.

The real four-component functions χ_1 and χ_2 undergo independently fast oscillations.
They become coupled only in the presence of an electromagnetic field.

$$\partial_t \chi_1 = \left(c\boldsymbol{\kappa}\cdot\nabla + \Omega\,\kappa^0\right)\chi_1 - \left(A_0 - \boldsymbol{\kappa}\cdot\boldsymbol{A}\right)\chi_2$$

$$\partial_t \chi_2 = \left(c\boldsymbol{\kappa}\cdot\nabla + \Omega\,\kappa^0\right)\chi_2 + \left(A_0 - \boldsymbol{\kappa}\cdot\boldsymbol{A}\right)\chi_1$$

$$\kappa^0 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad \kappa^1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\kappa^2 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \kappa^3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

EM potentials are measured in units of frequency $(e/\hbar)\mathrm{Volt} = 1.5 \times 10^{18}\mathrm{sec}^{-1}$ and the carrier frequency for electrons is $\boldsymbol{\Omega} = 7.76 \times 10^{20}\,\mathrm{sec}^{-1}$

Carrier frequency $\mathbf{\Omega} = 7.76 \times 10^{20}\,\mathrm{sec}^{-1}$ is huge.

There are no detectors that can follow such rapid vibrations. This extremely fast oscillation of the electronic carrier wave must be eliminated somehow to detect the signal.
We are not able to “listen” to this transmission because, our “ears” cannot follow the changes.

We cannot rectify the signal, like in the crystal radio. So, what can we do?
We must resort to some kind of homodyning.

In the theoretical description of this homodyning process **complex numbers** are very helpful. There are no problems on the **technical side** and we know very well how to proceed. There are two simple steps that lead to the Schrödinger equation. First, we form a complex combination from two real parts $\boldsymbol{\psi} = \chi_1 + i\chi_2$. Next, we extract the oscillations of the carrier wave from this complex wave function

$$\boldsymbol{\psi}(\boldsymbol{r}, t) = \exp(-i\Omega t)\Psi(\boldsymbol{r}, t)$$

The envelope wave function $\Psi(\boldsymbol{r}, t)$ is identified with the standard Schrödinger wave function.

The problem of fast oscillations seems to be gone but I am not at all happy with this result. What is missing is some understanding of these mathematical steps.

There are several crucial questions that remain unanswered.

I leave these questions with you as a summary of my talk.

Questions

Are the fast oscillations of the particle carrier wave **"elements of physical reality"**?

Are we neglecting some valid information about the system by removing these oscillations?

Is the measuring apparatus providing the local oscillator that "swallows" these oscillations?

Is the energy conservation of significance in this context?

What is the role of positive and negative frequencies?