Helicity in quantum electrodynamics

Once you identify it, you discover its presence everywhere

Iwo Bialynicki-Birula
Center for Theoretical Physics
Warsaw, Poland
www.cft.edu.pl/~birula

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- Helicity in particle physics

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- Helicity and the photon localization

Definition of helicity

Helicity is defined as the projection of the total angular momentum on the direction of momentum measured in units of $\hbar$

$$\boldsymbol{J} = \boldsymbol{r} \times \boldsymbol{p} + \boldsymbol{s} \qquad \hat{\chi} = \frac{\boldsymbol{J} \cdot \boldsymbol{p}}{p} = \boldsymbol{s} \cdot \boldsymbol{n} \qquad \boldsymbol{n} = \frac{\boldsymbol{p}}{p}$$

Spin matrices for spin-one particles in Cartesian coordinates ($\hbar = 1$)

$$s_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix} \quad s_y = \begin{bmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{bmatrix} \quad s_z = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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$$\hat{\chi}\, \boldsymbol{V} = i \begin{bmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = i \begin{bmatrix} n_y V_z - n_z V_y \\ n_z V_x - n_x V_z \\ n_x V_y - n_y V_x \end{bmatrix} = i \boldsymbol{n} \times \boldsymbol{V}$$

Helicity in coordinate space

Momentum operator is represented by the nabla

$$i\boldsymbol{p} \times \boldsymbol{V} \to \boldsymbol{\nabla} \times \boldsymbol{V}$$

Division by p is represented by a nonlocal kernel

$$\frac{1}{p} \to \int \frac{d^3k}{(2\pi)^3} \frac{e^{i\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')}}{k} = \frac{1}{2\pi^2|\boldsymbol{r}-\boldsymbol{r}'|^2}$$

$$\hat{\chi}\boldsymbol{V} = i\boldsymbol{n} \times \boldsymbol{V} \to \hat{\chi}\boldsymbol{V}(\boldsymbol{r}) = \int d^3r' \frac{1}{2\pi^2|\boldsymbol{r}-\boldsymbol{r}'|^2} \boldsymbol{\nabla} \times \boldsymbol{V}(\boldsymbol{r}')$$

Helicity operator is an involution in the subspace of divergenceless fields

$$\hat{\chi}^2\boldsymbol{V} = i\boldsymbol{n} \times (i\boldsymbol{n} \times \boldsymbol{V}) = \boldsymbol{V} - \boldsymbol{n}(\boldsymbol{n} \cdot \boldsymbol{V}) = \boldsymbol{V}$$

Helicity as a vector splitter

We can construct two projectors in the subspace of divergenceless fields

$$P_+ = \frac{1+\hat{\chi}}{2} \quad P_- = \frac{1-\hat{\chi}}{2}$$

$$P_+ + P_- = 1 \quad P_\pm^2 = P_\pm \quad P_+P_- = 0$$

With their use we can split any vector field into two parts:

The helicity +1 part $\boldsymbol{V}_+ = P_+\boldsymbol{V}$ and the helicity -1 part $\boldsymbol{V}_- = P_-\boldsymbol{V}$

Since the helicity operator is Hermitian the two parts are orthogonal

$$\int d^3r \boldsymbol{V}_-^*(\boldsymbol{r}) \cdot \boldsymbol{V}_+(\boldsymbol{r}) = 0$$

We will apply these results to electric and magnetic field vectors

Riemann-Silberstein vector

Riemann-Silberstein vector $\boldsymbol{F}$ is a complex combination of $\boldsymbol{D}$ and $\boldsymbol{B}$

$$\boldsymbol{F}(\boldsymbol{r},t) = \frac{\boldsymbol{D}(\boldsymbol{r},t)}{\sqrt{2\varepsilon}} + i\frac{\boldsymbol{B}(\boldsymbol{r},t)}{\sqrt{2\mu}}$$

It satisfies two Maxwell-like equations (free fields)

$$\partial_t \boldsymbol{F}(\boldsymbol{r},t) = -ic\boldsymbol{\nabla} \times \boldsymbol{F}(\boldsymbol{r},t) \quad \nabla \cdot \boldsymbol{F}(\boldsymbol{r},t) = 0$$

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The general solution of these equations

$$\boldsymbol{F}(\boldsymbol{r},t) = \int d^3k e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \left(\boldsymbol{e}(\boldsymbol{k})\alpha_+(\boldsymbol{k})e^{-i\omega t} + \boldsymbol{e}^*(\boldsymbol{k})\alpha_-(\boldsymbol{k})e^{i\omega t}\right)$$

The circular polarization vectors $\boldsymbol{e}(\boldsymbol{k})$ and $\boldsymbol{e}^*(\boldsymbol{k})$ are eigenvectors of $\hat{\chi}$

$$i\boldsymbol{k} \times \boldsymbol{e}(\boldsymbol{k}) = k\boldsymbol{e}(\boldsymbol{k}) \quad i\boldsymbol{k} \times \boldsymbol{e}^*(\boldsymbol{k}) = -k\boldsymbol{e}^*(\boldsymbol{k})$$

Splitting the field operator $\hat{\boldsymbol{F}}(\boldsymbol{r}, t)$

Projection operators $P_\pm$ split $\boldsymbol{F}(\boldsymbol{r}, t)$ into

Positive and negative frequency parts

Field operator $\hat{\boldsymbol{F}}(\boldsymbol{r}, t)$ is split into

Annihilation and creation parts

$$P_+\hat{\boldsymbol{F}}(\boldsymbol{r}, t) = \int d^3k \sqrt{\frac{\hbar kc}{(2\pi)^3}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \boldsymbol{e}(\boldsymbol{k}) \hat{a}_+(\boldsymbol{k}) e^{-i\omega t}$$

$$P_-\hat{\boldsymbol{F}}(\boldsymbol{r}, t) = \int d^3k \sqrt{\frac{\hbar kc}{(2\pi)^3}} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} \boldsymbol{e}(\boldsymbol{k}) \hat{a}_-^\dagger(\boldsymbol{k}) e^{i\omega t}$$

$P_\pm$ act only on the spatial coordinates but they determine time evolution

Splitting the field operators $\hat{\boldsymbol{D}}(\boldsymbol{r},t)$ and $\hat{\boldsymbol{B}}(\boldsymbol{r},t)$

Creation and annihilation parts of the electric and magnetic field operators

$$\hat{\boldsymbol{D}}(\boldsymbol{r},t) = (P_+ + P_-)\sqrt{\frac{\varepsilon}{2}}\left(\hat{\boldsymbol{F}}(\boldsymbol{r},t) + \hat{\boldsymbol{F}}^\dagger(\boldsymbol{r},t)\right) = \sqrt{\frac{\hbar c\varepsilon}{2}}\left[\hat{\boldsymbol{d}}^\dagger(\boldsymbol{r},t) + \hat{\boldsymbol{d}}(\boldsymbol{r},t)\right]$$

$$\hat{\boldsymbol{B}}(\boldsymbol{r},t) = (P_+ + P_-)\frac{1}{i}\sqrt{\frac{\mu}{2}}\left(\hat{\boldsymbol{F}}(\boldsymbol{r},t) - \hat{\boldsymbol{F}}^\dagger(\boldsymbol{r},t)\right) = \sqrt{\frac{\hbar c\mu}{2}}\left[\hat{\boldsymbol{b}}^\dagger(\boldsymbol{r},t) + \hat{\boldsymbol{b}}(\boldsymbol{r},t)\right]$$

$$\hat{\boldsymbol{d}}^\dagger(\boldsymbol{r},t) = \int\frac{d^3k}{(2\pi)^{3/2}}\sqrt{k}\left[\boldsymbol{e}^*(\boldsymbol{k})\hat{a}_+^\dagger(\boldsymbol{k}) + \boldsymbol{e}(\boldsymbol{k})\hat{a}_-^\dagger(\boldsymbol{k})\right]e^{ik\cdot x}$$

$$\hat{\boldsymbol{b}}^\dagger(\boldsymbol{r},t) = i\int\frac{d^3k}{(2\pi)^{3/2}}\sqrt{k}\left[\boldsymbol{e}^*(\boldsymbol{k})\hat{a}_+^\dagger(\boldsymbol{k}) - \boldsymbol{e}(\boldsymbol{k})\hat{a}_-^\dagger(\boldsymbol{k})\right]e^{ik\cdot x}$$

Helicity operator interchanges electricity and magnetism

$$\hat{\boldsymbol{d}}^\dagger = i\hat{\chi}\hat{\boldsymbol{b}}^\dagger \quad \hat{\boldsymbol{b}}^\dagger = -i\hat{\chi}\hat{\boldsymbol{d}}^\dagger$$

The analog of curl in mechanics

Harmonic oscillator in new uniform variables $\boldsymbol{q} = \hat{\omega}\boldsymbol{x}$

$$H(\boldsymbol{p}, \boldsymbol{q}) = \frac{\boldsymbol{p}^2}{2m} + m\frac{\boldsymbol{q}^2}{2} \qquad \{\boldsymbol{q}_i, \boldsymbol{p}_j\} = \omega_{ij}$$

$$\frac{d\boldsymbol{q}}{dt} = \frac{\hat{\omega}\boldsymbol{p}}{m} \qquad \frac{d\boldsymbol{p}}{dt} = -m\hat{\omega}\boldsymbol{q}$$

The matrix $\hat{\omega}$ is symmetric but it might be not necessarily positive

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In classical mechanics we can always write down the solutions

$$\boldsymbol{q}(t) = \boldsymbol{q}_0\cos(\hat{\omega}t) + \frac{\boldsymbol{p}_0}{m}\sin(\hat{\omega}t) \quad \boldsymbol{p}(t) = \boldsymbol{p}_0\cos(\hat{\omega}t) - m\boldsymbol{q}_0\sin(\hat{\omega}t)$$

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In quantum mechanics the situation is quite different since we need the positive square root $|\hat{\omega}|$ of $\hat{\omega}^2$ to construct energy eigenstates

$$\psi(\boldsymbol{x}) = (\text{Hermite polynomials in } \boldsymbol{x}) \times \exp\left(-\frac{m}{2\hbar}\boldsymbol{x}\cdot|\hat{\omega}|\cdot\boldsymbol{x}\right)$$

Use of helicity as the sign of curl

Transition rules from harmonic oscillator to electromagnetic field:

$$\boldsymbol{p} \longrightarrow -\boldsymbol{D} \quad \boldsymbol{q} \longrightarrow c\boldsymbol{B} \quad \boldsymbol{x} \longrightarrow c\boldsymbol{A} \quad m \longrightarrow \epsilon \quad \hat{\omega} \longrightarrow c\,\mathbf{curl}$$

$$H(\boldsymbol{p},\boldsymbol{q}) = \frac{\boldsymbol{p}^2}{2m} + m\frac{\boldsymbol{q}^2}{2} \longrightarrow H(\boldsymbol{D},\boldsymbol{B}) = \int d^3r \left(\frac{\boldsymbol{D}^2}{2\varepsilon} + \frac{\boldsymbol{B}^2}{2\mu}\right)$$

$$\{\boldsymbol{q}_i, \boldsymbol{p}_j\} = \omega_{ij} \longrightarrow \{B_i(\boldsymbol{r}), D_j(\boldsymbol{r}')\} = \epsilon_{ijk}\partial_k\delta^{(3)}(\boldsymbol{r}-\boldsymbol{r}')$$

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In quantum electrodynamics in order to replace $|\hat{\omega}|$ we need $|c\,\mathbf{curl}|$
To construct $|c\,\mathbf{curl}|$ we only need $\hat{\chi}$ — the sign of **curl**

$$|\hat{\omega}| = \mathrm{sign}\,(\hat{\omega})\,\hat{\omega} \to |c\,\mathbf{curl}| = c\,\mathrm{sign}(\mathbf{curl})\,\mathbf{curl} = c\,\hat{\chi}\,\mathbf{curl}$$

$$\exp\left(-\frac{m}{2\hbar}\boldsymbol{x}\cdot|\hat{\omega}|\cdot\boldsymbol{x}\right) \to \exp\left(-\frac{1}{2\hbar}\sqrt{\frac{\epsilon}{\mu}}\int d^3r\int d^3r'\,\frac{(\nabla\times\boldsymbol{A}(\boldsymbol{r}))\cdot(\nabla\times\boldsymbol{A}(\boldsymbol{r}'))}{2\pi^2|\boldsymbol{r}-\boldsymbol{r}'|^2}\right)$$

The electromagnetic field is one infinite-dimensional harmonic oscillator

The Wigner function

The Wigner function for the ground state of a harmonic oscillator is obtained from the wave function by a simple Gaussian integration

$$\begin{aligned} W(\boldsymbol{p},\boldsymbol{x}) &= \int d^n\eta\, \psi(\boldsymbol{x}-\boldsymbol{\eta})\psi^*(\boldsymbol{x}+\boldsymbol{\eta})e^{2i\boldsymbol{p}\cdot\boldsymbol{\eta}/\hbar} \\ &= \exp\left[-\frac{1}{\hbar}\left(\frac{\boldsymbol{p}\cdot|\hat{\omega}^{-1}|\cdot\boldsymbol{p}}{m}+m\boldsymbol{x}\cdot|\hat{\omega}|\cdot\boldsymbol{x}\right)\right] \end{aligned}$$

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We need one more transition rule

$$|\hat{\omega}^{-1}| \to (\hat{\chi}\mathtt{curl})^{-1} = \left(\delta_{ij}-\frac{\partial_i\partial_j}{\Delta}\right)\frac{1}{2\pi^2|\boldsymbol{r}-\boldsymbol{r}'|^2}$$

Thus the Wigner function for the electromagnetic field is

$$\exp\left[-\frac{1}{2\pi^2\hbar c}\int d^3r\int d^3r'\left(\frac{\boldsymbol{D}(\boldsymbol{r})\cdot\boldsymbol{D}(\boldsymbol{r}')}{\varepsilon|\boldsymbol{r}-\boldsymbol{r}'|^2}+\frac{\boldsymbol{B}(\boldsymbol{r})\cdot\boldsymbol{B}(\boldsymbol{r}')}{\mu|\boldsymbol{r}-\boldsymbol{r}'|^2}\right)\right]$$

Reasonable definition of the photon localization

Def: Photon is localized in some region R if all measurements of the electromagnetic field outside of R do not reveal the photon's existence

We can introduce two separate notions:
magnetic localization and electric localization

A photon is in a magnetically localized state at time t if the expectation values of all products of magnetic fields evaluated in this state

$$\langle 1\mathrm{ph}|\hat{B}_i(\boldsymbol{r}_1,t)\hat{B}_j(\boldsymbol{r}_2,t)\dots\hat{B}_k(\boldsymbol{r}_n,t)|1\mathrm{ph}\rangle$$

have their vacuum values when all points $\boldsymbol{r}_i$ lie outside of R
Analogous definition holds for electric localization

It turns out that we can achieve at most only
an instantaneous magnetic or electric localization

Helicity operator prevents complete localization

If a photon is magnetically localized in R the expectation value $\langle 1\mathrm{ph}|\hat{B}_i(\boldsymbol{r}_1,t)\hat{B}_j(\boldsymbol{r}_2,t)|1\mathrm{ph}\rangle$ is equal to its vacuum value outside of R

To achieve this we assume that φ in $|1\mathrm{ph}\rangle = \int d^3r\, \boldsymbol{\varphi}(\boldsymbol{r})\cdot\boldsymbol{d}^\dagger(\boldsymbol{r})|0\rangle$ is properly normalized and sharply localized in R

The expectation value in question can be computed

$$\langle 1\mathrm{ph}|\hat{B}_i(\boldsymbol{r}_1,t)\hat{B}_j(\boldsymbol{r}_2,t)|1\mathrm{ph}\rangle = \langle 0|\hat{B}_i(\boldsymbol{r}_1,t)\hat{B}_j(\boldsymbol{r}_2,t)|0\rangle$$

$$+\phi_i^*(\boldsymbol{r}_1,t)\phi_j(\boldsymbol{r}_2,t)+\phi_i(\boldsymbol{r}_1,t)\phi_j^*(\boldsymbol{r}_2,t) \qquad \boldsymbol{\phi}(\boldsymbol{r},t) = \mathrm{const}\,\boldsymbol{\nabla}\times\boldsymbol{\varphi}(\boldsymbol{r},t)$$

It follows from the $\boldsymbol{d}\leftrightarrow\boldsymbol{b}$ symmetry (confirmed by a direct computation) that to achieve also the electric localization $\hat{\chi}\boldsymbol{\phi}$ must be localized
This cannot happen since for a localized $\boldsymbol{\phi}$ the $1/r^2$ tail extends to infinity

$$\hat{\chi}\boldsymbol{\phi}(\boldsymbol{r},t) = \int\frac{d^3r'}{2\pi^2|\boldsymbol{r}-\boldsymbol{r}'|^2}\boldsymbol{\nabla}\times\boldsymbol{\phi}(\boldsymbol{r}',t)$$

What kind of localization is possible?

Not only we cannot have simultaneous magnetic and electric localization but even one of those types is highly fragile — it only lasts for one instant

In a magnetically localized photon state its electric field is everywhere
Owing to the Maxwell equations the magnetic field will be immediately produced everywhere and the initial magnetic localization will be ruined

In the absence of a complete sharp localization we may only try for the "next to the best" that we will call an ϵ-localization

A family of photon states is ϵ-localized in R if for every ϵ there exists a member state such that the departure of the field expectation values from the vacuum values is less than ϵ for all points outside of R
Such states can be found among the eigenstates of the helicity operator

$$\begin{aligned}
\boldsymbol{F}(\boldsymbol{r},t) &= \nabla\times\left[(i/c)\partial_t\mathbf{Z}(\mathbf{r},t)+\nabla\times\mathbf{Z}(\mathbf{r},t)\right]\\
\boldsymbol{Z}(\boldsymbol{r},t) &= \boldsymbol{m}\frac{l}{ir}\left[\exp\left(-2\sqrt{1+i(ct-r)/l}\right)-\exp\left(-2\sqrt{1+i(ct+r)/l}\right)\right]
\end{aligned}$$

Summary

- It is worthwhile to introduce the helicity operator in QED
- In position space helicity is a nonlocal operator
- Most of the nonlocalities found in QED can be attributed to the helicity operator