

Radiative decay of Trojan wave packets

Zofia Białynicka-Birula

Institute of Physics, Lotników 32/46, 02-668 Warsaw, Poland

Iwo Białynicki-Birula

Center for Theoretical Physics, Lotników 32/46, 02-668 Warsaw, Poland

(Received 16 June 1997)

We calculate the decay rates due to spontaneous emission for electronic states described by Trojan wave packets. The spontaneous decay rate for a typical Trojan state ($n = 60$) is six orders of magnitude smaller than the ionization rate and it is about one order of magnitude smaller than the rate for the corresponding ordinary circular Rydberg state. [S1050-2947(97)07411-8]

PACS number(s): 31.50.+w, 32.70.Fw, 32.80.-t

It has been recently shown theoretically [1–6] that in highly excited atoms placed in a strong, resonant circularly polarized electromagnetic wave there exist dressed electronic states in the form of nonstationary, nonspreading wave packets, called Trojan wave packets [1]. These states are highly stable against ionization [5] with a lifetime of about 10^6 Kepler periods for wave packets corresponding to $n = 60$. In the present paper we calculate the decay rate of Trojan wave packets due to spontaneous emission and show that this process is much slower than the ionization. The concept of Trojan states has been further generalized [7–11] by adding the magnetic field to achieve an even stronger stabilization of the wave packets. However, in the present paper we do not consider the influence of the magnetic field.

In order to perform the calculation of the lifetime, we have to generalize the standard method based on the Fermi golden rule, as shown in [12], to deal with transitions between nonstationary, rotating states. The dynamics of Trojan wave packets interacting with the quantized electromagnetic field is governed by the Hamiltonian

$$H(t) = \frac{\mathbf{p}^2}{2m} - \frac{e^2}{4\pi\epsilon_0 r} - e\mathcal{E}(x \cos \Omega t + y \sin \Omega t) - e\mathbf{r} \cdot \mathbf{E} + H_{\text{field}}, \quad (1)$$

where \mathcal{E} is the amplitude of the driving, circularly polarized wave propagating in the z direction, \mathbf{E} denotes the electric field operator

$$\mathbf{E}(\mathbf{r}) = \sum_{J,M,\lambda} \int_0^\infty d\omega \sqrt{\frac{\hbar\omega}{2\epsilon_0}} \mathbf{E}^{(\lambda)}(J, M, \omega; \mathbf{r}) a_{JM\lambda}(\omega) + \text{H.c.}, \quad (2)$$

and H_{field} is the free-field Hamiltonian

$$H_{\text{field}} = \sum_{J,M,\lambda} \int_0^\infty d\omega \hbar\omega a_{JM\lambda}^\dagger(\omega) a_{JM\lambda}(\omega). \quad (3)$$

We use the expansion of the electric-field operator into the spherical waves to carry through the transition to the rotating frame [12]. The operators $a_{JM\lambda}(\omega)$ annihilate photons with the frequency ω and the quantum numbers J (total angular

momentum of the photon), M (z component of the angular momentum), and λ [electric ($\lambda = e$) or magnetic ($\lambda = m$) type]. All calculations will be done in the dipole approximation; the electric fields in Eq. (1) are evaluated at the center of the atom.

In order to calculate the rate of spontaneous emission using the standard perturbation theory, we eliminate (cf. Ref. [12]) the time dependence of the Hamiltonian by the time-dependent unitary transformation $\exp(i\Omega M_z t)$, where M_z is the z component of the total angular momentum operator

$$M_z = xp_y - yp_x + \sum_{J,M,\lambda} \int_0^\infty d\omega \hbar M a_{JM\lambda}^\dagger(\omega) a_{JM\lambda}(\omega). \quad (4)$$

Under this transformation the Schrödinger equation $i\partial_t |\psi\rangle = H |\psi\rangle$ undergoes the change

$$i\partial_t |\tilde{\psi}\rangle = \tilde{H} |\tilde{\psi}\rangle, \quad (5)$$

where

$$|\tilde{\psi}\rangle = e^{i\Omega M_z t} |\psi\rangle, \quad (6)$$

and

$$\tilde{H} = \frac{\mathbf{p}^2}{2m} - \frac{e^2}{4\pi\epsilon_0 r} - e\mathcal{E}x - e\mathbf{r} \cdot \mathbf{E} + H_{\text{field}} - \hbar\Omega M_z. \quad (7)$$

The electronic part of this Hamiltonian

$$H_{\text{el}} = \frac{\mathbf{p}^2}{2m} - \frac{e^2}{4\pi\epsilon_0 r} - e\mathcal{E}x - \Omega(xp_y - yp_x), \quad (8)$$

describes the electron dressed by the driving electromagnetic wave. Since H_{el} is time independent, it has stationary eigenstates $|\phi_E\rangle$

$$H_{\text{el}} |\phi_E\rangle = E |\phi_E\rangle. \quad (9)$$

Transitions between these eigenstates will be accompanied by emission of photons due to the coupling $-e\mathbf{r} \cdot \mathbf{E}$ to the quantized electromagnetic field.

The Trojan states are not strictly the eigenstates of H_{el} , but they are rather long-lived resonances. Their wave functions are well localized near the $z=0$ plane and also in the radial and the azimuthal variables on this plane. Due to their spatial localization they can be effectively described [1,4,11] in the harmonic approximation when the Coulomb potential is expanded around the classical equilibrium point x_0 ,

$$-e^2/4\pi\epsilon_0x_0^2 + e\mathcal{E} + mx_0\Omega^2 = 0. \quad (10)$$

The results based on such an expansion depend on the choice of the coordinate system. It has been shown in [2] that the harmonic approximation in the polar coordinates leads to wave packets that spread much more slowly than the original Trojan wave packets [1] obtained in the Cartesian coordinates because they much more closely approximate the ‘‘true’’ Trojan wave packets obtained by the exact numerical diagonalization [4,5] of the Hamiltonian (8). Nevertheless, we shall use here the simplest Trojan wave packets (in Cartesian coordinates) to calculate the spontaneous decay rate since the improved wave functions would not change any of the prefactors that determine the rate but would only slightly modified the values of the relevant dipole matrix elements.

In the harmonic approximation the Hamiltonian H_{el} can be expressed in terms of creation and annihilation operators [1,13]

$$H_{el} \approx H_Q = \hbar\omega_+ b_+^\dagger b_+ - \hbar\omega_- b_-^\dagger b_- + \hbar\omega_z b_z^\dagger b_z + E_0, \quad (11)$$

where

$$\omega_\pm = \Omega \sqrt{2 - q \pm \sqrt{9q^2 - 8q}/\sqrt{2}}, \quad (12)$$

$$\omega_z = \Omega \sqrt{q}, \quad (13)$$

$$q = e^2/4\pi\epsilon_0 m \Omega^2 x_0^3. \quad (14)$$

For the stability of the Trojan wave packets [1], the parameter q must lie in the range (8/9,1); the optimal value is $q=0.9562$.

The fundamental state, annihilated by b_+ , b_- , and b_z , is described by the Gaussian wave function

$$\psi_0 = N_0 e^{im\Omega x_0 y/\hbar} e^{-iE_0 t/\hbar} \times e^{-m\Omega[A(x-x_0)^2 + By^2 + 2iC(x-x_0)y + Dz^2]/2\hbar}, \quad (15)$$

where

$$A = \sqrt{(1+2q)[4f(q) - 9q^2]}/3q, \quad (16)$$

$$B = \sqrt{(1-q)[4f(q) - 9q^2]}/3q, \quad (17)$$

$$C = f(q)/3q, \quad D = \sqrt{q}, \quad (18)$$

$$f(q) = 2 + q - 2\sqrt{(1-q)(1+2q)}. \quad (19)$$

The state $|\psi_0\rangle$, having the energy E_0 , is the best localized Trojan state, but it is not the state with the lowest energy because of the inverted sign in the second term in H_Q . Therefore, it can decay into the state $|\psi_1\rangle = b_-^\dagger |\psi_0\rangle$ with the eigenenergy $E_0 - \omega_-$, emitting a photon. The emitted pho-

ton undergoes the rotational frequency shift, as we have shown recently [12], because it is radiated by a nonstationary, rotating state of the electron. Therefore, the argument of the energy-conservation δ -function in the formula for the transition rate γ_T is shifted by ΩM ,

$$\gamma_T = \frac{2\pi}{\hbar^2} \int_0^\infty d\omega |\langle \psi_1, 1_{\not{z}} | e\mathbf{r} \cdot \mathbf{E} | \psi_0, 0_{\not{z}} \rangle|^2 \delta(\omega - \omega_- - \Omega M). \quad (20)$$

According to Eq. (12), the energy difference ω_- is always smaller than Ω ; hence the negative values of M are excluded by energy conservation. In the dipole approximation the emitted photon has its orbital angular momentum equal to 0 and we are left with only one possible value $M=1$. This means that the photon polarization matches that of the circularly polarized driving wave, but the photon frequency is different. Thus the final state of the photon is $a_{11}^\dagger(\omega)|0_{\not{z}}\rangle$ and the photonic matrix element appearing in Eq. (20) is

$$\langle 1_{\not{z}} | \mathbf{E}(0) | 0_{\not{z}} \rangle = \frac{1}{2\pi} \sqrt{\frac{\hbar\omega^3}{3\epsilon_0 c^3}} (1, -i, 0). \quad (21)$$

The final state of the electron has the wave function

$$\psi_1 = N_1 [\mu(x-x_0) + iy] e^{im\Omega x_0 y/\hbar} e^{-i(E_0 - \omega_-)t/\hbar} \times e^{-m\Omega[A(x-x_0)^2 + By^2 + 2iC(x-x_0)y + Dz^2]/2\hbar}, \quad (22)$$

where

$$N_1 = N_0 \frac{\sqrt{2m\Omega/\hbar}}{\sqrt{\mu^2/A + 1/B}}, \quad (23)$$

$$\mu = (1+C)/(A + \omega_-/\Omega). \quad (24)$$

Hence the relevant component of the electronic-dipole element is equal to

$$\langle \psi_1 | [(x-x_0) - iy] | \psi_0 \rangle = \sqrt{\frac{\hbar}{2m\Omega}} \frac{\mu/A - 1/B}{\sqrt{\mu^2/A + 1/B}}. \quad (25)$$

The decay rate depends on the Kepler frequency Ω and the parameter q only,

$$\gamma_T = \frac{e^2}{4\pi\epsilon_0 mc^3} \frac{\Omega^2}{3} \frac{(\mu/A - 1/B)^2}{\mu^2/A + 1/B} (1 + \omega_-/\Omega)^3. \quad (26)$$

For the optimal value of $q=0.9562$ the spontaneous decay rate is

$$\gamma_T = 5.27 \frac{e^2}{4\pi\epsilon_0 mc^3} \Omega^2 = 4.95 \times 10^{-23} \Omega^2 \text{ s}. \quad (27)$$

When the Trojan wave packet is localized near the circular orbit $n=60$, the Kepler frequency is $2 \times 10^{11} \text{ s}^{-1}$ and we obtain

$$\gamma_T = 1.98 \text{ s}^{-1}, \quad n=60. \quad (28)$$

The spontaneous decay rate per one Kepler period $T=1/2\pi\Omega$ is 1.58×10^{-12} . Thus, it is about six orders of

magnitude smaller than the ionization rate. The smallness of the decay rate justifies our use of approximate wave functions since any improvement will not change the main conclusion that the spontaneous decay rate of the Trojan states cannot compete with other decay mechanisms.

The spontaneous decay rate for Trojan states is about one order of magnitude smaller than the spontaneous decay rate γ_R of the circular Rydberg state $n=60$ ($\gamma_R=14\text{ s}^{-1}$). Circular Rydberg states are known to be exceptionally stable against decay; therefore, it is worth stressing that the Trojan wave packets are even more stable. Their stability relative to that of Rydberg states increases with n since the rates γ_T and γ_R scale differently with n . The rate γ_T as a function of n ,

$$\gamma_T = \frac{8.85 \times 10^{10}}{n^6(1+1/2n)^3} \text{ s}^{-1}, \quad (29)$$

is obtained from Eq. (26) after expressing Ω^2 from Eq. (14) and by identifying x_0 with the average value $\langle r \rangle$ of the radial variable for hydrogenic states [14]

$$\langle r \rangle = a_0[3n^2 - l(l+1)]/2. \quad (30)$$

The rate γ_R ,

$$\gamma_R = \frac{1.071 \times 10^{10}(1-1/n)^{2n-2}}{n^5(1-1/2n)^{4n-1}} \text{ s}^{-1}, \quad (31)$$

can be calculated from the formulas given in Ref. [14]. The two rates γ_T and γ_R become almost equal for the lowest possible value $n=2$. This is much below the region where Trojan wave packets may form. The rate γ_T is exactly ten times smaller than γ_R for $n=80$. Thus the Trojan state, a

nonstationary, coherent superposition of several Rydberg states dressed by a strong resonant electromagnetic wave, is, quite surprisingly, more stable against spontaneous decay than the stationary, bare Rydberg state.

Although a Trojan wave packet represents a quantum state of an electron, it closely resembles a classical charge cloud moving on a large circular orbit. Therefore, one may expect that the decay rates will also be similar in those two cases. In the classical case, we may define this rate as the relative decrease of the electron energy in time

$$\gamma_{cl} = \frac{1}{E} \frac{dE}{dt}. \quad (32)$$

The power radiated by a moving pointlike electron is governed (in the nonrelativistic regime) by the Larmor formula [15]

$$\frac{dE}{dt} = \frac{2}{3} \frac{e^2}{4\pi\epsilon_0 c^3} |\dot{\mathbf{v}}|^2. \quad (33)$$

Thus, for an electron in a circular motion with an angular frequency Ω , we obtain

$$\gamma_{cl} = \frac{4}{3} \frac{e^2}{4\pi\epsilon_0 m c^3} \Omega^2. \quad (34)$$

The only difference between this result and our result (27) for a Trojan state is in the value of the numerical coefficient. However, they are of the same order of magnitude, in agreement with the principle of correspondence.

This work has been supported by KBN Grant No. 2P30B04313.

-
- [1] I. Bialynicki-Birula, M. Kalinski, and J. H. Eberly, *Phys. Rev. Lett.* **73**, 1777 (1994).
 [2] M. Kalinski, J. H. Eberly, and I. Bialynicki-Birula, *Phys. Rev. A* **52**, 2460 (1995).
 [3] D. Farrelly and T. Uzer, *Phys. Rev. Lett.* **74**, 1720 (1995).
 [4] D. Delande, J. Zakrzewski, and A. Buchleitner, *Europhys. Lett.* **32**, 107 (1995).
 [5] J. Zakrzewski, D. Delande, and A. Buchleitner, *Phys. Rev. Lett.* **75**, 4015 (1995).
 [6] M. Kalinski and J. H. Eberly, *Phys. Rev. A* **53**, 1715 (1996).
 [7] E. Lee, A. F. Brunello, and D. Farrelly *Phys. Rev. Lett.* **75**, 3641 (1995).
 [8] D. Farrelly, E. Lee, and T. Uzer, *Phys. Lett. A* **204**, 359 (1995).
 [9] A. F. Brunello, T. Uzer, and D. Farrelly, *Phys. Rev. Lett.* **76**, 2874 (1996).
 [10] E. Lee, A. F. Brunello, and D. Farrelly, *Phys. Rev. A* **55**, 2203 (1997).
 [11] C. Cerjan, E. Lee, D. Farrelly, and T. Uzer, *Phys. Rev. A* **55**, 2222 (1997).
 [12] I. Bialynicki-Birula and Z. Bialynicka-Birula, *Phys. Rev. Lett.* **78**, 2539 (1997).
 [13] I. Bialynicki-Birula and Z. Bialynicka-Birula, *Phys. Rev. Lett.* **77**, 4298 (1996).
 [14] H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Springer, Berlin, 1957).
 [15] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975).