Collective excitations of electrons by an intense wave in a magnetic field

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Collective variables describing the dipole and the quadrupole oscillations of the system of charged particles are introduced, and their equations of motion are analyzed in the presence of a constant magnetic field. The magnetic field causes shifts and splittings of the characteristic frequencies of the collective oscillations. In the neighborhood of these frequencies, parametric resonance occurs when the system is irradiated by an electromagnetic wave. In the regions of parametric resonance, the energy transferred to the system of particles grows exponentially in time.

INTRODUCTION

Collective excitations in many-particle systems are of great importance in many areas of physics, most notably in nuclear physics, in plasma physics, and in solid-state physics. Collective excitations in atomic physics were first predicted theoretically more than 50 years ago by Bloch and later confirmed theoretically and found experimentally in the form of the atomic giant dipole resonance in one-photon (high-frequency) spectroscopy. Recently, the problem of collective atomic excitations came into focus in connection with the experiments on the multiple ionization of atoms by intense laser beams.

In our previous paper, we used the model of the atom (nicknamed the “pseudoatom” by Moshinsky et al.) with harmonic forces to study the role of collective excitations in the energy transfer from the wave field to the atom. In the present paper, we extend our analysis to include the interaction with a constant magnetic field. We will study the collective motions of the system in terms of the following set of collective operators:

The total Hamiltonian of the system has the form

\[
H = \frac{1}{2m} \sum_n \int d^3r \psi_n^+ (r, t) [(p - eA)^2 + (\mathbf{k} \cdot \mathbf{r})^2] \psi_n (r, t) - \frac{1}{4} \mathbf{k}^2 \int d^3r \int d^3r' \psi_n^+ (r, t) \psi_n (r, t) \times (r - r')^2 \psi_n^+ (r', t) \psi_n (r', t),
\]

where \( \psi_n (r, t) \) and \( \psi_n^+ (r, t) \) denote the anticommuting field operators that annihilate and create an electron at the point \( r \) with the spin projection \( \sigma \). The anisotropic harmonic potential is described by a diagonal matrix \( \mathbf{K} \):

\[
\mathbf{K} = \text{diag}[K_{11}, K_{22}, K_{33}].
\]

We will choose the potential of the constant magnetic field \( \mathbf{B} \) in the symmetric gauge,

\[
\mathbf{A} = \frac{1}{\mu}(\mathbf{B} \times \mathbf{r}).
\]

The second part of the Hamiltonian [Eq. (1)] describes repulsive harmonic forces among the particles, whose strength is measured by the constant \( \mathbf{k}^2 \).

We will study the collective motions of the system in terms of the following set of collective operators:

The total number of particles

\[
N = \int \psi_n^+ (r, t) \psi_n (r, t),
\]

The position of the center of mass of the particle cloud

\[
\mathbf{R} = \int \psi_n^+ (r, t) \mathbf{r} \psi_n (r, t),
\]

The total momentum

\[
\mathbf{P} = \int \psi_n^+ (r, t) \mathbf{r} \psi_n (r, t),
\]

and
that describe the collective quadrupole degrees of freedom.

In Eqs. (4)–(9) the integral sign denotes both the integration over the position \( r \) and the summation over \( \sigma \).

The Hamiltonian [Eq. (1)] can be expressed in terms of the collective operators in the following way:

\[
H_D = \frac{1}{2mN} \left[ P^2 + (\mathbf{R} \cdot \mathbf{R})^2 \right] - \frac{e}{2mN} \mathbf{B} \cdot (\mathbf{R} \times \mathbf{P}) + \frac{e^2}{2m^2N} (\mathbf{B} \times \mathbf{R})^2,
\]

where \( \hat{\kappa} \) is a diagonal matrix of the form

\[ \hat{\kappa} = \text{diag}[(K_{11})^2 - \hbar^2 N, (K_{22})^2 - \hbar^2 N, (K_{33})^2 - \hbar^2 N], \]

and \( U_T \) denotes the transverse trace of the matrix \( U \), i.e., the sum of the diagonal elements in the plane perpendicular to the magnetic field \( \mathbf{B} \). In Eq. (12), the convention to sum over repeated indices is adopted.

Our collective operators form a closed commutator algebra; their only nonvanishing commutators are

\[
[R_i, P_j] = i\delta_{ij}N, \]  
\[
[U_{ij}, T_{kl}] = \frac{1}{2}i [\delta_{ik}W_{jl} + \delta_{jl}W_{ik} + \delta_{jl}W_{ki} + \delta_{ij}W_{lk}], \]  
\[
[U_{ij}, W_{kl}] = \frac{1}{2}i [\delta_{il}U_{jk} + \delta_{jl}U_{ik}], \]  
\[
[T_{ij}, W_{kl}] = -\frac{1}{2}i [\delta_{il}U_{jk} + \delta_{jl}U_{ik}], \]  
\[
[W_{ij}, W_{kl}] = \frac{1}{2}i [\delta_{il}W_{jk} - \delta_{jl}W_{ki}].
\]

It follows from these commutation relations that the oscillations of the dipole degrees of freedom, described by \( \mathbf{R} \) and \( \mathbf{P} \), are independent of the quadrupole oscillations, described by the remaining 21 operators \( U_{ij}, T_{ij}, \) and \( W_{ij} \).

We present below the full set of equations of motion for the collective operators. Even though these equations can be written for an arbitrary orientation of the magnetic field, we assume, for simplicity, that the harmonic potential has cylindrical symmetry and that the magnetic field has the direction of the cylinder's axis (chosen as the third axis of our coordinate system). We choose the system of units in which \( \hbar = 1 = c \) and \( m = 1 \).

The equations of motion for the dipole operators are

\[
\dot{R}_1 = P_1 + \beta R_1, \quad \dot{R}_2 = P_2 - \beta R_1, \quad \dot{R}_3 = P_3,
\]

where

\[
\beta = eB/2, \quad \omega_L = (K_{11})^2 + \beta^2)^{1/2}.
\]

As seen from the above equations, the dipole oscillates in exactly the same fashion as a single particle under the combined influence of the harmonic and magnetic forces. Its characteristic frequencies are

\[
\omega_L, \quad \omega_T + \omega_C/2, \quad \omega_T - \omega_C/2,
\]

where \( \omega_C \) is the cyclotron frequency,

\[
\omega_C = eB/mc = 2\beta.
\]

The equations of motion for the quadrupole operators are

\[
\begin{align*}
\dot{U}_{11} &= 2W_{11} + 2\beta U_{12}, \\
\dot{U}_{22} &= 2W_{22} - 2\beta U_{12}, \\
\dot{U}_{12} &= W_{12} + W_{21} - \beta U_{11} - U_{22}, \\
\dot{T}_{11} &= -2\mu W_{11} - 2\beta T_{12}, \\
\dot{T}_{22} &= -2\mu W_{22} - 2\beta T_{12}, \\
\dot{T}_{12} &= -\mu(W_{12} + W_{21}) - \beta(T_{11} - T_{22}), \\
\dot{W}_{11} &= T_{11} - \mu U_{11} + \beta(W_{12} + W_{21}), \\
\dot{W}_{22} &= T_{22} - \mu U_{22} - \beta(W_{12} + W_{21}), \\
\dot{W}_{12} &= W_{12} - \mu U_{12} - \beta(W_{11} - W_{22}), \\
\dot{U}_{33} &= 2W_{33}, \\
\dot{T}_{33} &= -2\kappa W_{33}, \\
\dot{W}_{33} &= T_{33} - \kappa U_{33}, \\
\dot{U}_{13} &= W_{13} + W_{31} + \beta U_{23}, \\
\dot{U}_{23} &= W_{23} + W_{32} - \beta U_{13}, \\
\dot{T}_{13} &= -\mu W_{13} - \kappa W_{31} + \beta T_{23}, \\
\dot{T}_{23} &= -\mu W_{23} - \kappa W_{32} - \beta T_{13}, \\
\dot{W}_{13} &= T_{13} - \kappa U_{13} + \beta W_{23}, \\
\dot{W}_{31} &= T_{31} - \mu U_{13} + \beta W_{32}, \\
\dot{W}_{23} &= T_{23} - \kappa U_{33} - \beta W_{13}, \\
\dot{W}_{32} &= T_{32} - \mu U_{23} - \beta W_{31},
\end{align*}
\]

where

\[
\kappa = (K_{33})^2 - \hbar^2 N, \\
\beta = (K_{11})^2 - \hbar^2 N + (2\beta)^2
\]

The task of finding the characteristic frequencies of the collective quadrupole oscillations is made simpler by the decoupling of the three sets of components, as indicated in Eqs. (19)–(21). The eigenfrequencies obtained from the three characteristic equations corresponding to these sets of equations are

\[
\omega_C, \quad \Omega_T, \quad \Omega_T \pm \omega_C \quad [\text{for Eqs. (19)}], \\
\omega_L \quad [\text{for Eqs. (20)}], \\
\omega_L \quad [\text{for Eqs. (20)}].
\]
where
\[
\Omega_T = 2 \mu L = 2 [(K_{11})^2 - k^2 N + \omega_c^2]^{1/2},
\]
(24a)
\[
\Omega_L = 2 \mu L = 2 [(K_{33})^2 - k^2 N]^{1/2}.
\]
(24b)

It is worth observing that, in the absence of the mutual interaction between the particles, all the quadrupole frequencies can be obtained as sums and differences of the fundamental frequencies, i.e., the dipole frequencies, as was to be expected. Mutual repulsive forces decrease these fundamental frequencies, as is seen from Eqs. (24). This leads to a softening of the collective quadrupole oscillations.

### INTERACTION WITH THE ELECTROMAGNETIC WAVE

The physical significance of various modes of collective oscillations can be established by coupling the system to an external perturbation such as an electromagnetic wave. In the presence of an electromagnetic wave, the Hamiltonian \( H \) has the same form [Eq. (1)], but the vector potential \( A \) is now a sum of two terms,
\[
A = \frac{1}{2}(B \times r) + A_{\text{wave}}(r, t).
\]
(25)

In the general case, such a perturbation mixes all the modes of the oscillations. For simplicity, we consider only a linearly polarized, plane, monochromatic wave described by the potential
\[
A_{\text{wave}}(r, t) = a_L \epsilon \sin(\omega_L t - k \cdot r).
\]
(26)

If the dimensions of the system are much smaller than the wavelength, as is often the case, we can resort to a multipole expansion and keep only the lowest terms. The dipole and the quadrupole terms in the interaction Hamiltonian are
\[
H_I = -ea_L \epsilon \cdot P \sin(\omega_L t) - \frac{1}{2}(ea_L)^2 N \sin^2(\omega_L t)
\]
\[
- \frac{1}{2}(ea_L)^2 k \cdot R \sin(2\omega_L t)
\]
\[
+ ea_L N^{-1}(k \cdot R)(\epsilon \cdot P)\cos(\omega_L t)
\]
\[
+ 2ea_L k \cdot \hat{W} \cdot \epsilon \cos(\omega_L t).
\]
(27)

Even though the quadrupole terms are usually smaller, they are important, because they influence the time evolution of the system in a qualitatively different way. They lead to the parametric coupling, which in the region of parametric resonance can dramatically change the rate of the energy transfer to the system.

In the dipole approximation, only the collective dipole is driven by the external force, as described by the first term of the interaction Hamiltonian [Eq. (27)]. If the wave frequency \( \omega_L \) is equal to one of the dipole frequencies (17), an ordinary resonance occurs, and the system can absorb energy at the rate growing linearly with time.

In the quadrupole approximation, both the dipole and the quadrupole oscillations are affected by the wave through the parametric coupling. In this case the parametric resonance may occur in certain regions in the amplitude-frequency plane \((a_L, \omega_L)\) of the electromagnetic wave. In those regions the energy of the system grows exponentially with time.

### DISCUSSION

We have shown above that the system of charged particles moving in a harmonic potential and in a constant magnetic
field exhibits parametric resonance when coupled to electromagnetic radiation. We have analyzed this resonance in terms of collective variables describing the total dipole and the total quadrupole of the system. One obvious application of this analysis is to the study of the many-electron atoms irradiated by strong laser light. In principle, the magnetic field allows one to tune the atom to the frequency of the laser. Unfortunately, the magnetic fields available in the laboratory, of the order of 10 T, are not sufficient to produce significant shifts and splittings of the frequencies in the optical region. However, we do believe that the role of the magnetic field in collective excitations of a system of charged particles may be of importance in those cases when the characteristic frequencies are comparable to the cyclotron frequency.

In the extreme case, when there are no harmonic forces in the plane perpendicular to the magnetic field, our analysis should apply to the quantized Hall effect. Then, only the harmonic potential along the third axis is needed to keep the electrons at the surface, and the magnetic field is the only cause of the orbit quantization in the directions perpendicular to the field.

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