

## Representativeness, majority rule and the square root law

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As a founding stone for a construction of the voting system for the Council of the EU one should take the *representativeness principle*, based on the following two rules: the *equitability rule* - the indirect voting power of all citizens of the EU should be as nearly equal as possible, and the *majority rule* - the decisions of the Council should express as closely as possible the will of the majority of the population of the EU.

The equitability rule is realized (under some natural assumptions), if the assumption of the *Penrose square root law* is fulfilled: *the voting power of each country behaves proportionally to the square root of population*. How to construct a system fulfilling the majority rule?

To illustrate the problem, consider a model ‘union’ consisting of one large state (*G*) with population of 49 millions, three medium (*D,E,F*) with population 16 million each, and three small (*A,B,C*) with 1 million citizens each. For simplicity, assume that the double majority system (DM) and the Penrose system (P) are based on the simple majority of 50% of weights (population for DM, square root of population for P) and states (DM). If the polarisation of opinion on a given issue is as in the table below, only 45% of the population of the ‘union’ is in favour of the legislative. However, under the rules of the double majority system the decision is taken against the will of this vast majority. The difference 10m between 55m (those who disagree with the decision) and 45m (who agree) is called the *majority deficit*. When the Penrose system is applied, the coalition gains only 10 votes and fails to gather the required majority. Hence, in this case, the decision of the Council and the will of the majority of the citizens of the EU coincide.

State	A	B	C	D	E	F	G	total	
Population [m]	1	1	1	16	16	16	49	100	
Votes <b>Yes</b> [m]	0.67	0.67	0.66	6	6	6	25	<b>45</b>	Majority deficit = 55 – 45 = <b>10</b>
Votes <b>No</b> [m]	0.33	0.33	0.34	10	10	10	24	<b>55</b>	
Member state votes (DM)	1	1	1	0	0	0	1	4 (out of 7)	= <b>Yes</b>
Population votes (DM)	1	1	1	0	0	0	49	52 (out of 100)	= <b>Yes</b>
Square root weights	1	1	1	4	4	4	7	22	
Square root votes (P)	1	1	1	0	0	0	7	10 (out of 22)	= <b>No</b>

Although in the above case all the quota were set to 50%, it is not difficult to construct analogous examples for the rules used in the Constitution system, which requires 55% of members states and 65% of the population. The *Morriss square root law* (1987) states that (again under some natural assumptions) these examples can be further generalized: *the mean majority deficit is minimal for the voting system with the weights proportional to the square root of the population* [M87, FM99]. To understand this result consider the minister representing the largest country *G* with population of 49 millions. In the double majority system he uses his 49 votes against the will of 24 millions of inhabitants. On the other hand, the minister of the small state *A* can misrepresent at most half a million of his compatriots. In other words the precision of determining the will of all the citizens is larger in smaller states, and so the votes of their ministers should gain a proportionally higher weight, which is the case in the Penrose system.

In summary, voting systems with the square root weights are optimal with respect to both the *equitability* and *majority* principles. Hence such voting systems can be considered as more representative and democratic, and in this sense better for the entire union: in particular, they prevent politicians to take the decision against the will of the majority of citizens of the EU.

M87: P. Morriss (1987), *Power — A Philosophical Analysis*, Manchester University Press. Manchester.

FM99: D. S. Felsenthal, M. Machover (1999), Minimizing the mean majority deficit: The second square-root rule. *Mathematical Social Sciences* 37, 25-37.