Jagiellonian Compromise
an alternative voting system for
the Council of the European Union

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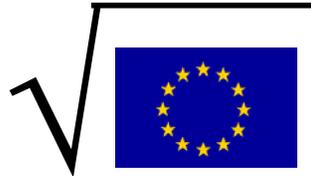
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*In this paper, which forms an expanded version of our previously published articles, we propose an **alternative compromise solution for the distribution of votes in the Council of the European Union**, free of the defects characterising both the Treaty of Nice and the Treaty Establishing a Constitution for Europe voting systems.*

'I cannot conceive of the Community

without total parity'

Konrad Adenauer to Jean Monnet at a meeting held in Bonn on 4th April 1951 during the preparations leading up to the signing of the 'Treaty Establishing the European Coal and Steel Community'

1. Voting weights, powers and indices

Voting rules implemented by various political or economic bodies may be studied with the help of tools developed for many decades in game theory [Felsenthal, Machover 1998]. A mathematical theory of indirect voting was initiated after World War II by a British psychiatrist and mathematician, Lionel S. Penrose, in the context of a hypothetical distribution of votes in the United Nations General Assembly [Penrose 1946]. He introduced the concept of *a priori* **voting power**, a quantity measuring the ability of a participant of the voting body to influence the decisions taken. This notion may be also used for analysing rules governing decision taking in the Council of the European Union (EU).

It is important to clearly differentiate here between the **voting weight** of a given country and its potential **voting power**, the latter reflecting the extent to which it may influence decisions taken by the Council when all possible coalitions between different countries are taken into consideration. To illustrate the difference with just a simple example: a shareholder with 51% of stocks of a company has only 51% of all votes (voting weight) at the shareholders' assembly, but he takes 100% of the voting power if the assembly takes decisions by a simple majority vote. In other words, *a Member State's power is defined as its capacity to affect EU Council **decisions** and not merely to affect Council **votes**. In this perspective, when a member State's vote does not affect the decision, it is not considered an expression of its relative power* [Bobay 2001].

Clearly, with 27 Member States and potentially complicated voting procedures, it is a non-trivial task to analyse the distribution of power in the Council since one has to consider more than 134 million of possible coalitions. To quantify the notion of voting power, mathematicians introduced the concept of **power index** which measures the potential voting power of each member of the voting body. Although the current scientific literature contains several competing definitions of power indices, the original Penrose concept is often used. In his approach, the *a priori* voting power of a country is proportional to the probability that its vote will be decisive in a hypothetical ballot: should this country decide to change its vote, the winning coalition would fail

to satisfy the qualified majority condition. Without any further information about the voting body it is natural to assume that all potential coalitions are equally likely. This very assumption leads to the concept of **Penrose-Banzhaf index** (PBI, or simply **Banzhaf index)** called so after John Banzhaf, an American attorney, who introduced this index independently in 1965 [Banzhaf 1965]. For convenience Banzhaf indices are often normalised in such a way that their sum is equal to unity. It is easy to show that the voting power held by a given country depends not only on its voting weight but also on the distribution of the weights among all the remaining Member States of the European Union.

Note that this approach is purely normative, not descriptive: we are interested in the potential voting power arising from the voting procedure itself. Although in the mathematical model it is assumed that all the coalitions are equally likely, in reality some coalitions are *a priori* more probable than others. Thus, the actual voting power depends on the polarisation of opinion in the voting body and may change from voting to voting. Dennis Leech from the University of Warwick referred to this problem in the following terms: *An understanding of where power lies requires us to take account of many relevant factors: the political complexions of governments, the Paris-Bonn axis, the commonality among the Benelux countries, the Nordic or Mediterranean members, the small states versus the large states, new Europe versus old Europe, the Eurozone, etc. etc. […] But from the point of view of the design of the formal voting system in a union that is expanding with the admission of new members being quite a normal process, it would clearly be inappropriate to base constitutional parameters like voting weights on such considerations. […] That would appear arbitrary and would fail to provide a guide for what the votes of new entrants should be. Far better to allocate the voting weights on the basis of general philosophical principles that can be seen to apply equally to all countries and citizens, to new members as well as old ones. A priori power indices are useful in this* [Leech 2003]. Definitely, the model of the calculation of voting power based on the counting of majority coalitions is applicable while analysing institutions in which alliances are not permanent, but change depending upon the nature of the matter under consideration. The Council of the European Union constitutes just such a body.

It is often claimed during discussions on voting power that the Banzhaf index can only reflect the power of a particular country to form a blocking coalition within the EU Council. In fact, the indices reflecting the power to block a decision (so called **Coleman preventive power index**) and the power to form a coalition capable of forcing a decision (so called **Coleman initiative power index**) are both mutually proportional and depend proportionally on the Banzhaf index. The ratios of proportionality depend on the decision rule, nevertheless, under any decision rule, if one country has more preventive power than another, then it also has more initiative power. Thus, both phenomena constitute simply the two sides of the same coin.

The meaning of voting power is not purely theoretical. In a series of papers Mika Widgrén from the Turku School of Economics and Heikki Kauppi from the University of Helsinki investigate the relationship between the Council voting rules and EU budget transfers by using a power politics model [Kauppi, Widgrén 2004, 2006, Widgrén 2006]. The main claim of the authors is that the distribution of voting power in the Council alone is sufficient to explain most of budget allocation: *[…] quantitative power indices stemming from voting theory provide a good description of the actual distribution of power among EU members. Of course, we cannot directly verify the accuracy of such indices since it is impossible to measure power directly. Instead, we evaluate whether these power measures explain a quantifiable manifestation of the exercise of power, namely members' shares of EU budget allocation. […] when we apply specific voting power measures that allow correlated preferences and cooperative voting patterns between the member states, our estimates indicate that the power politics view can explain as much as 90% of the budget shares. We conclude that power politics can explain a major part of the Council decisions* [Kauppi, Widgrén 2004].

2. Two voting systems: Nice vs. Constitution

Pursuant to the Treaty of Nice (2001) each Member State is assigned a voting weight which to some degree reflects its population. The Council adopts a piece of legislation if:

(a)   the sum of the weights of the Member States voting in favour is at least 255 (with the sum of the weights of all 27 Member States being 345) which is approximately 73.9 %;

(b)   a majority of Member States (i.e. at least 14 out of 27) vote in favour;

(c)   the Member States forming the qualified majority represent at least 62 % of the overall population of the European Union.

For a proposal to pass, all three of these conditions must be satisfied, and so the system should therefore be thought of as one requiring a '**triple majority**'. However, as the mathematical analysis has shown, condition (a) is the most significant one, since the probability of forming a coalition which would meet only this condition and fail to meet the other two is extremely low [Felsenthal, Machover 2001].

According to the agreement reached in Brussels in June 2004 and signed in Rome in October 2004, the Council of Ministers of the European Union acting on a proposal from the Commission or from the Union Minister for Foreign Affairs takes its decisions if two criteria ('**double majority**') are simultaneously satisfied:

(b)   at least 55% of Member States vote in favour;

(c)   these Member States comprise at least 65% of the overall population of the European Union.

   Additionally:

(d) a blocking minority must include at least four Council members, failing which the qualified majority shall be deemed attained. The same rules apply to the European Council when it is acting by a qualified majority.

As can be seen, the Constitutional Treaty removes condition (a): adding the weights of individual votes. To put it differently, the weights applied are directly proportional to the population of each individual Member State.

Majority of experts agree that both the voting system established by the Treaty of Nice (2001) and the Treaty Establishing a Constitution for Europe (2004) have obvious drawbacks.

It seems that the basic defect of the Nice voting rules lies in the low **decision-making efficiency** of the system. This efficiency, also called the **Coleman power of a collectivity to act**, is measured as the probability that the Council would approve a randomly selected issue and it equals to the proportion of winning coalitions assuming that all coalitions are equally likely. Whereas in the case of the rules laid down in the Constitutional Treaty approximately 12.9% of possible coalitions lead to approving a randomly selected issue, for the Treaty of Nice this index is equal to only 2.1%. However, according to the recent analysis by Sebastian Kurpas and Justus Schönlau from the Centre for European Policy Studies in Brussels this deficiency has only theoretical meaning: *Innovations affecting the Council are the ones cited most frequently when arguments in favour of the Constitutional Treaty are made. One central element was supposed to be the new rules for the weighting of member states' votes under qualified majority voting, which were designed to reduce the danger of blockage in a more diverse Union. Calculations have proved that the introduction of the 'double majority system' would indeed make it much easier to avoid the formation of blocking minorities with 25 member states. So far, however, figures on the voting reality do not confirm the widespread fear of deadlock after enlargement while the Nice system of weighting votes is still in place* [Kurpas, Schönlau 2006], see also [Hagemann, De Clerck-Sachsse 2007, Trzaskowski 2007]. On the other hand, it is possible to show that with no fundamental change in the voting power of each particular Member State, the Nice system may be modified so that its formal effectiveness increases significantly, see [Baldwin, Berglöf, Giavazzi, Widgrén 2000, Baldwin, Widgrén 2004].

Another defect of the Nice system lies in the necessity to apply simultaneously three criteria in the calculation of the qualified majority. However, the studies conducted by Dan S. Felsenthal and Moshé Machover from the London School of Economics and Political Science have shown that with a slight correction of the weights assigned to each country one can achieve a system almost equivalent to the Nice one but much simpler, as it is based on the weight criterion alone [Felsenthal, Machover 2001]. Therefore, the rules underlying the Nice system may be cut ruthlessly with Ockham's razor, as the mechanism based on the three criteria is needlessly complex and difficult to comprehend by the average citizen of the Union.

Although the proposal contained in the Constitutional Treaty does away with the voting weights, which have no objective basis, and constitutes a more efficient voting system, it has flaws of its own. Many authors pointed out that it is favourable to the most and to the least populated countries at the expense of all medium size states. For instance Richard Baldwin from the Graduate Institute of International Studies in Geneva and Mika Widgrén from the Turku School of Economics summarised their analysis of the Constitutional Treaty voting rules in the following words: *The Constitutional Treaty rules will break the traditional French German power equality making France the junior partner in the Franco-German alliance. Spain and Poland will lose the Near-Big status they won in the Nice Treaty. The biggest losers are the Mediums (about 10 million citizens). The chief winners are the Big-4; Germany alone wins more than the other Big-4 combined* [Baldwin, Widgrén 2004].

In fact, all voting systems in the decision-making bodies of the European Union, and formerly of the European Communities, have been based on a compromise between two principles: the principle of **equality of Member States** and that of **equality of citizens**, and the 'double majority' system apparently reflects these principles. However, in this system large states would gain a lot from the direct link to population, while small countries would derive disproportionate power from the increase in the number of states needed to support a proposal. The combined effect would sap influence away from medium-sized countries. Ironically, a similar conclusion follows from a book written fifty years earlier by Lionel Penrose, who discovered this drawback of a 'double majority' system, writing as early as 1952:

If two votings were required for every decision, one on a per capita basis and the other upon the basis of a single vote for each country, this system would be inaccurate in that it would tend to favour large countries,

see [Penrose 1952]. This passage may be interpreted as a prophetic and critical opinion on the arrangements laid down in the Constitutional Treaty, formulated 50 years before their adoption by the Intergovernmental Conference of the European Union in Brussels in 2004. We shall see that the arrangements adopted at Nice distribute the influence upon the decision-making process in the Council among all the citizens of the Union more evenly than those proposed in the Constitution.

The fact that the Constitutional Treaty makes use of only two criteria does not remedy another of its basic defects: the system is not transparent since an average citizen has no simple way of calculating the potential voting power held by each Member State under this system. This requires equally complex mathematical calculations as under the Nice system. Such calculations show that the basic democratic principle, that the vote of any citizen of any Member State is of equal worth, is violated in both systems.

3. Jagiellonian compromise

A. Square root weights

Is it possible to objectively design a voting system, in which each citizen of each Member State would have the same power to influence the decisions made on his or her behalf? Can it be done in a way that is transparent, easy to implement, efficient to use, and will readily accommodate any future extensions of the EU? We believe that the answer is 'yes'.

A partial response to this question was already given by Penrose who deliberated principles of an ideal representative voting system in which **each citizen of each country has the same potential voting power**. First consider direct elections of the government (which nominates the minister voting on behalf of the entire country in the Council) in a state with population N. It is easy to imagine that an average German citizen has smaller influence on the election of his government than, for example, a citizen of the neighbouring Luxembourg. Penrose proved, under some natural assumption, that in such elections the voting power of a single citizen decays as one over the square root of N. Thus, the system of indirect voting applied to the Council is representative in the above sense, if **the voting power of each country behaves proportionally to the square root of** N, so that both factors cancel out. This statement is known in the literature under the name of the **Penrose square root law**. Kauppi and Widgrén express this claim in the following words: *[…] although using a square-rooted population as the basis for a voting scheme might sound mysterious, it can also be justified from the point of view of fairness. It can be shown that in a two-tier decision-making system (e.g. the Member States at the lower level and the EU at the upper) the square-root rule guarantees under certain circumstances that each citizen is equally represented in the Council regardless of his/her home country. This proposal obtained considerable support from academics* [Kauppi, Widgrén 2004].

Some people may find this counter-intuitive and think that the weights (or powers) should be directly proportional to population. However, as it was observed by Edward Best from the European Institute of Public Administration in Maastricht: *It has never*

*been the case that the principle underlying the representation of states in the EU system qua states is that of direct proportionality. […] the 'Founding Fathers' of Europe explicitly rejected 'objective' keys and population, in favour of a distribution of votes reflecting a balancing act between states. This balance was conceived in terms of clusters of states and responded to a general principle of **'degressive proportionality'** (which figures explicitly in the text of the draft Constitution with regard to the European Parliament) by which the larger units are under-represented compared to the smaller ones* [Best 2004].

The idea of dividing votes proportionally to the square root of population not only has a special position in the mathematical theory of voting, but is in fact the simplest mathematical implementation of the principle of degressive proportionality and lies exactly between two extremes: 'one country-one vote' (as if the European Union were a loose association of states) and votes proportional to population (as if the European Union were a single state). Note that a similar degressive system is also used in the German Bundesrat to assign the number of representatives to each Land.
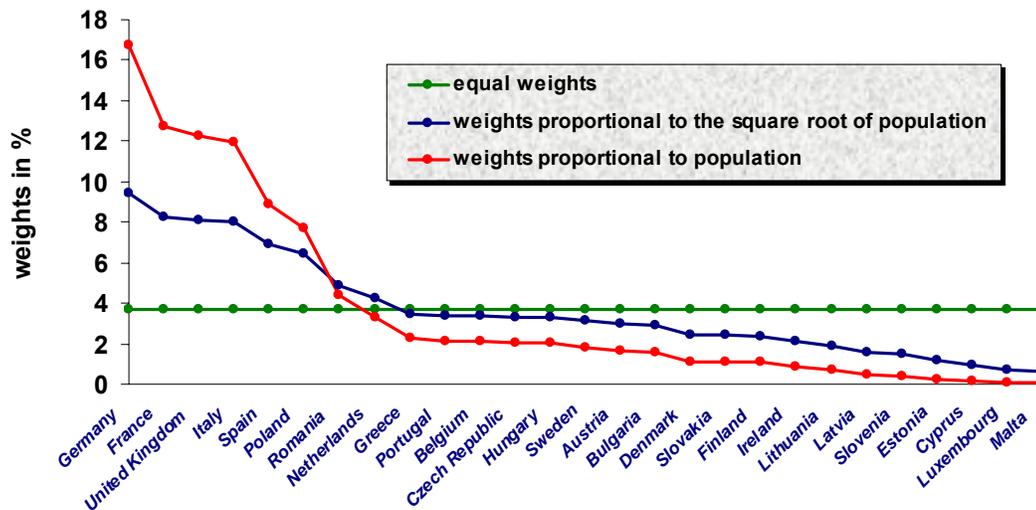

Fig. 1. The weights attributed to the Member States: proportionally to population, proportionally to the square root of population, and uniformly.

A voting system based on the Penrose square-root law was first proposed for the Council of Ministers in 1996 by Annick Laruelle of the Université Catholique de Louvain in Belgium and Mika Widgrén of the Turku School of Economics and Business Administration in Finland. Since then it has been analysed by many authors from different countries, including: Laruelle, Widgrén (1998), Baldwin, Berglöf, Giavazzi, Widgrén (2000), Felsenthal, Machover (2000-2004), Hosli (2000), Sutter (2000), Tiilikainen, Widgrén (2000), Kandogan (2001), Leech (2002), Moberg (2002), Hosli, Machover (2002), Leech, Machover (2003), Widgrén (2003), Baldwin, Widgrén (2004), Bilbao (2004), Bobay (2004), Kirsch (2004), Lindner (2004), Lindner, Machover (2004), Plechanovová (2004, 2006), Sozański (2004), Ade (2005), Koornwinder (2005), Pajala (2005), Maaser, Napel (2006), Taagepera, Hosli (2006) and others. Such voting procedures have also been used in practice in several international institutions. Prior to the European Union summit in Brussels in June 2004, an open letter in support of square-root voting weights in the Council of Ministers endorsed by more than 40 scientists from 10 European countries had been sent to EU institutions and the governments of the Member States.

When a system based on the square root weights was put forward by Swedish diplomats in 2000, Sweden's Prime Minister Göran Persson said: *Our formula has the advantage of being easy to understand by public opinion and practical to use in an enlarged Europe […] it is transparent, logical and loyal.* The former Irish Prime Minister John Bruton also made numerous positive references to voting systems based on the Penrose law in 2004: *Instead of double majority, we could put in the Treaty a new, clear and automatic mathematical formula for allocating voting weights. Such a formula has been proposed by researchers in the London School of Economics. Their formula would allocate voting weights to countries on the basis of the square root of their population, rather than the number of population itself.*

On the other hand, the **square root voting system** was proposed independently in the EU context **without** any relation to the Penrose square root law by several authors: Schmitter & Torreblanca (1997) (under the name **proportional proportionality**); Moberg (1998), Bovens, Hartmann (2002), Mabille (2003), and Beisbart, Bovens, Hartmann (2004).

Fig. 2. The 'square root' weights attributed to Member States are proportional to the sides of the squares representing their populations: exemplary data for seven selected countries of different size.

The assertion that 'the voting power of each country should be proportional to the square root of its population' does not entirely solve the problem. Werner Kirsch from Ruhr-Universität Bochum explained this as follows: *The square root law tells us how the power should be distributed among the countries. It is, however not clear at a first glance how to implement it in terms of voting weights, as the voting weights do not give the power indices immediately* [Kirsch 2004]. Accordingly, the question arise: how to allocate weights and how to set majority quota or threshold (the Council reaches a decision when the sum of the weights of the Member States voting in favour exceeds that threshold) to obtain required distribution of power. The answer we have proposed is surprisingly simple: **one should choose the weights to be proportional to the square root of the population and then find such an optimal quota that would produce the maximally transparent system, that is, a system under which the voting power of each Member State would be approximately equal to its voting weight** [Życzkowski, Słomczyński 2004].

B. Optimal quota for qualified majority

The choice of an appropriate decision-taking quota (threshold) affects both the distribution of voting power in the Council (and thus also the representativeness of the system) and the voting system's effectiveness and transparency. Different authors have proposed different quotas for a square root voting systems, usually varying from 60% to 74%.

In a series of papers we have shown that it is possible to find **the optimal quota** enabling the computed voting power of each country to be practically equal to the attributed voting weight, and so to be proportional to the square root of the population [Życzkowski, Słomczyński, Zastawniak 2006, Słomczyński, Życzkowski 2006, 2007], see also [Feix, Lepelley, Merlin and Rouet 2007]. Hence the Penrose law is practically fulfilled, and the potential influence of every citizen of each Member State on the decisions taken in the Council is the same (representativeness). Such a voting system is not only **representative** but also **transparent**: the voting powers are proportional to the voting weights. Furthermore, the system is **simple** (one criterion only) and **objective**: it does not *a priori* favour nor handicap any European country. It has been dubbed the '**Jagiellonian Compromise**' by the media.

For the **Council of Ministers of EU-27** the optimal quota equals **61.6%** (while it was 62% for EU-25). For an arbitrary voting body the optimal quota *q* can be approximated by a simple mathematical formula:

$$q = \frac{1}{2}\left(1 + \frac{\sqrt{N_1 + \ldots + N_M}}{\sqrt{N_1} + \ldots + \sqrt{N_M}}\right) \qquad \textbf{(Q)}$$

where N_i stands for the population of the *i*-th country.

Since the number of Member States is not going to be explicitly provided in the text of the European Constitution, defining specific threshold for the qualified majority should be avoided. In the system under consideration the optimal quota depends on the number of members of the voting body and their size, so there should be a possibility

to adjust it in future without modifying the European Constitution. Due to the above formula the system proposed is **easily extendible**.

The detailed calculation shows that the optimal quota decreases with the size of the voting body. This choice of the quota guarantees that the system is **moderately efficient** with the efficiency always larger than 15.9%, which is even more than in the Constitutional Treaty, not to mention the Nice system. Besides, the efficiency of the system we propose does not decrease with increasing number of Members States, whereas the efficiency of the 'double majority' system does.

The representative voting system based on the Penrose square root law and the appropriate choice of optimal quota may be used as a reference point while analysing the rules established by politicians. The table and figure below present a comparison of the voting power (measured by the PBI) of EU members according to the voting system established by the Treaty of Nice, that proposed in the Constitutional Treaty, and the 'Jagiellonian compromise' (square root weights plus the optimal quota equal 61.6%).

The 'Jagiellonian compromise', which allocates voting power according to the square root of population, restores some of the power to medium-sized countries (from Spain to Ireland) that would be taken away by the Constitution. It is also apparent why it is called a compromise. Germany, for example, would gain considerable power under the new system compared with the Treaty of Nice, but not as much as it would if the proposals in the Constitution were adopted. It is important to stress that similar conclusions can be achieved by examining how other indices of the potential voting power change.

There has been a long tradition of weighted voting in the Council of European Union (or earlier in the Council of the European Communities). As politicians have agreed in the past to voting weights allotted somewhat arbitrarily, they could find even more acceptable the voting weights allotted according to an explicit rule based on clear principles.
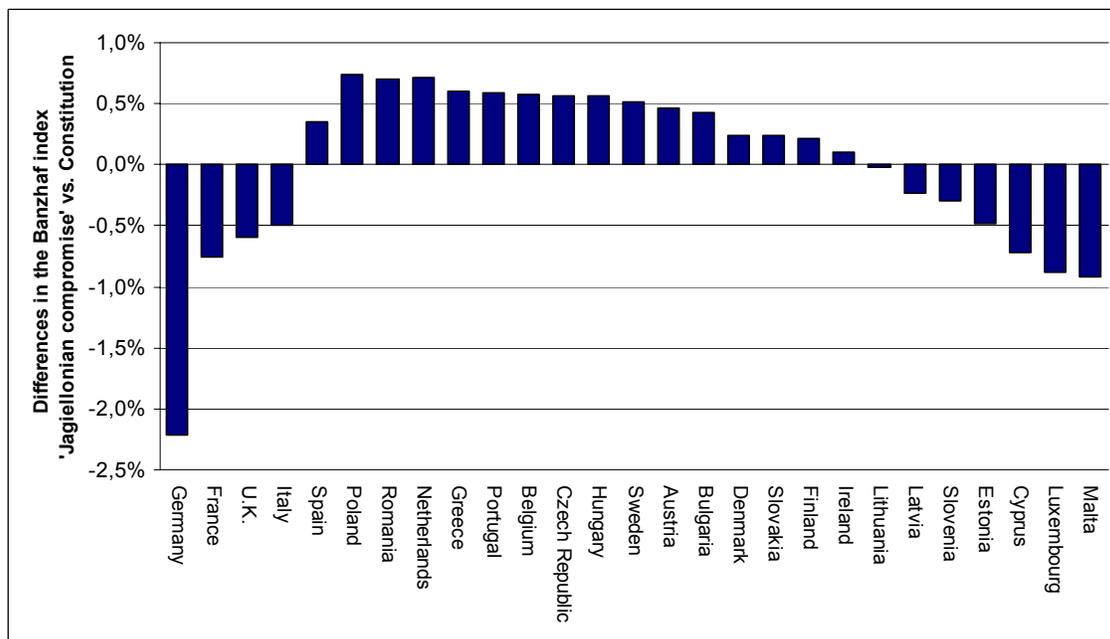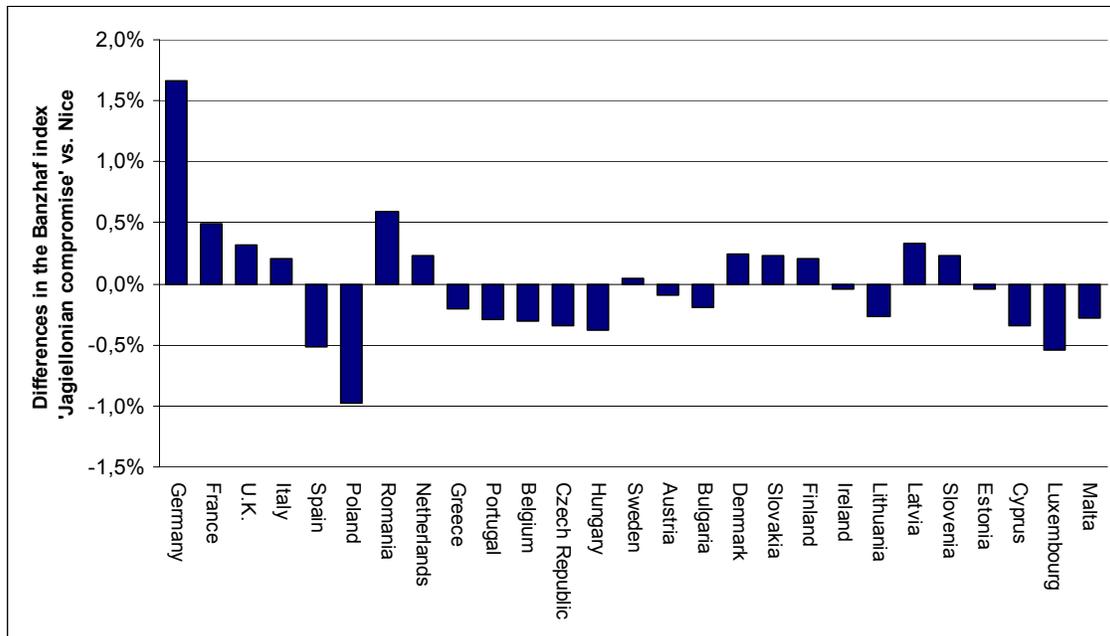
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Fig. 3. If the 'Jagiellonian compromise' was adopted, the voting power for each of the 27 EU states would change. The top graph shows the differences relative to the existing system in the Treaty of Nice and the bottom graph relative to that in the proposed Constitution. The countries are ordered by population from largest to smallest. The calculations are based on data from: Lanzieri G. Population in Europe 2005: first results. *EUROSTAT*. Statistics in focus. Population and social conditions 2006, 16: 1-12.

Jagiellonian Compromise for EU-27: $q = 61.6$

Member State	Population (in millions)	Voting power (Constitution)	Voting weight (JC)	Voting power (JC)
Germany	82.44	11.66	9.47	9.45
France	62.89	9.02	8.27	8.27
United Kingdom	60.39	8.69	8.10	8.10
Italy	58.75	8.49	7.99	7.99
Spain	43.76	6.55	6.90	6.91
Poland	38.16	5.71	6.44	6.45
Romania	21.61	4.15	4.85	4.85
Netherlands	16.33	3.50	4.21	4.21
Greece	11.13	2.88	3.48	3.48
Portugal	10.57	2.80	3.39	3.39
Belgium	10.51	2.80	3.38	3.38
Czech Republic	10.25	2.77	3.34	3.34
Hungary	10.08	2.74	3.31	3.31
Sweden	9.05	2.63	3.14	3.14
Austria	8.27	2.53	3.00	3.00
Bulgaria	7.72	2.47	2.90	2.90
Denmark	5.43	2.19	2.43	2.43
Slovakia	5.39	2.18	2.42	2.42
Finland	5.26	2.17	2.39	2.39
Ireland	4.21	2.04	2.14	2.14
Lithuania	3.40	1.95	1.92	1.92
Latvia	2.29	1.81	1.58	1.58
Slovenia	2.00	1.78	1.48	1.48
Estonia	1.34	1.69	1.21	1.21
Cyprus	0.77	1.63	0.91	0.91
Luxembourg	0.46	1.59	0.71	0.71
Malta	0.40	1.58	0.66	0.66

Tab. 1. Comparison of voting power of EU-27 member states: population; voting power measured by PBI in the system of the European Constitution; voting weights and voting power measured by PBI in the proposed solution ('Jagiellonian Compromise') based on the Penrose law with the 61.6% threshold.

4. Conclusions

We shall conclude this paper by proposing a complete voting system based on the Penrose square root law. The system consists of a single criterion only and is determined by the following two rules:

A. The voting weight attributed to each Member State is proportional to the square root of its population;

B. The decision of the voting body is taken if the sum of the weights of members of a coalition exceeds the 61.6% quota. In each case of a future extension of the Union the value of the quota is adjusted according to the formula (Q).

We would like to emphasize the following advantages of the proposed voting system:

- it is extremely **simple** since it is based on a single criterion, and thus it could be called a '**single majority**' system;

- it is **objective** (no arbitrary weights or thresholds), hence it cannot *a priori* handicap any member of the European Union;

- it is **representative**: every citizen of each Member State has the same potential voting power;

- it is **transparent**: the voting power of each Member State is (approximately) proportional to its voting weight;

- it is **easily extendible**: if the number of Member States changes, all that needs to be done is to set the voting weights according to the square root law and adjust the quota accordingly;

- it is **moderately efficient**: as the number of Member States grows, the efficiency of the system does not decrease;

- it is also **moderately conservative**, that is, it does not lead to a dramatic transfer of voting power relative to the existing arrangements.

From a pragmatic point of view the square root system proposed here is as easy to use as the double majority system, for which fulfilment of the population criterion during a given voting will in practice be confirmed with the help of a calculator. A voting system for use in the European Council is likely to be treated as a model solution to be used later while designing voting rules for other international institutions.

The European Union is known to be eager to foster modern technology and promote scientific research. Therefore, while designing the European Constitution it would be natural to make use of results of game theory and operation research conducted in Europe in the last 50 years.

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