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Random Lasers

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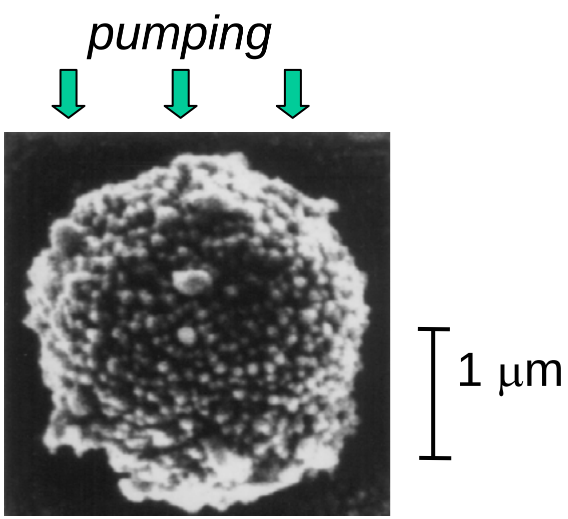
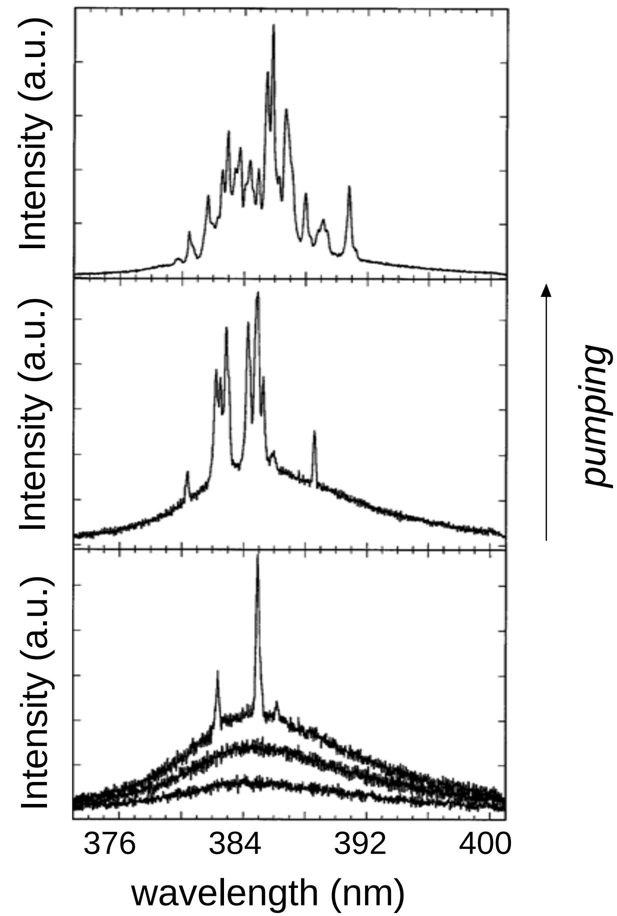
Random Lasers

H. Cao et al., 1999:

Disordered ZnO-clusters and films





feedback by chaotic scattering of light

Theoretical Ideas

Lethokov: 1967

– photon diffusion + linear gain

John, Wiersma & Lagendijk: 1995--

– photon & atomic diffusion
– non-linear coupling

Beenakker & coworkers: 1998--

– random scattering approach
– focus on linear regime below laser threshold
– rate equations for strongly confined chaotic resonator

Soukoulis & coworkers: 2000--

– numerical simulation of time-dependent Maxwell-equations coupled to active medium

issues to be discussed here:

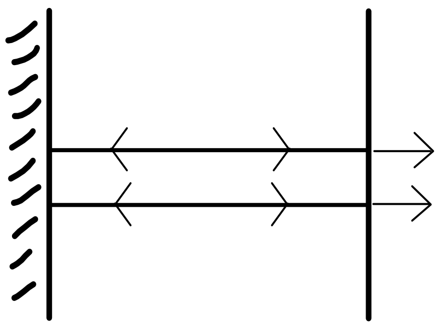
- quantization for bad chaotic /disordered resonators

- increase of laser linewidth for bad resonators

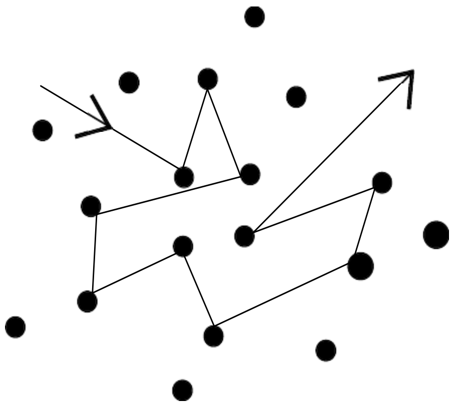
- statistics for # of laser peaks

- statistics for # of emitted photons

Why modifications to laser theory?



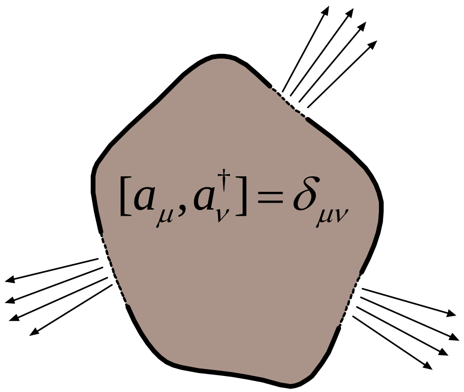
- modes have simple spatial form
- approximation: eigenmodes of closed resonator



- modes have complex spatial distribution
- spectrally overlapping

field quantization for open resonators

PRL ***89****, 083902 (2002),*
PRA ***67****, 013805 (2003)*
J. Opt. B ***6****, 211 (2004)*



$$[b_m(\omega), b_{m'}^\dagger(\omega')] = \delta_{mm'}\,\delta(\omega - \omega')$$

system ↓ *bath* ↓

$$H = \sum_\mu \hbar\omega_\mu a_\mu^\dagger a_\mu + \sum_m \int d\omega\, \hbar\omega\, b_m^\dagger(\omega) b_m(\omega)$$

$$+\hbar \sum_\mu \sum_m \int d\omega [W_{\mu m}(\omega) a_\mu^\dagger b_m(\omega) + V_{\mu m}(\omega) a_\mu b_m(\omega) + h.c.]$$

drop antiresonant terms!

$V_{\mu m}(\omega), W_{\mu m}(\omega)$: Integrals of mode functions along the openings

Dynamics: Heisenberg picture

damping *noise*

$$\dot{a}_\mu = -i\omega_\mu a_\mu - \sum_\nu \kappa_{\mu\nu} a_\nu + F_\mu \qquad \kappa = \pi W W^\dagger$$

standard laser:

open laser:

master equation

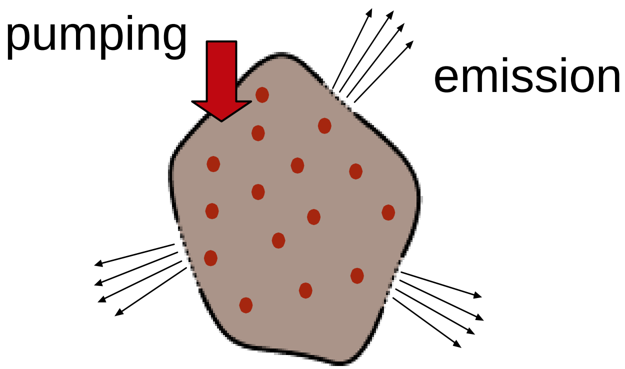
Schrödinger picture, low temperature: $k_B T < \hbar\omega$

$$\dot{\rho} = -\frac{i}{\hbar}[\sum_{\mu} \hbar\omega_{\mu} a_{\mu}^{\dagger} a_{\mu}, \rho]$$

$$+\sum_{\mu\nu} \kappa_{\mu\nu} \{[a_{\mu}, \rho a_{\nu}^{\dagger}] + [a_{\nu}\rho, a_{\mu}^{\dagger}]\}$$

G. Hackenbroich et al., PRA 013805 (2003): generalization of single-mode master equation to many overlapping modes

open laser



field-atom coupling (spatially random for chaotic modes)

non-linear coupled Heisenberg equations

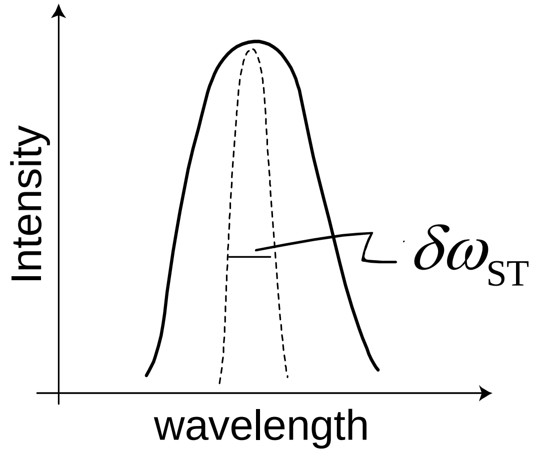
$\dot{a}_\mu = \ldots$ field modes

$\dot{S}_{-p} = \ldots$ polarization of p-th atom

$\dot{S}_{zp} = \ldots$ inversion of p-th atom

modes coupled not only via atoms but also through damping

increase of laser linewidth



regime of a single laser oscillation

$$\delta\omega = K\,\delta\omega_{ST}$$

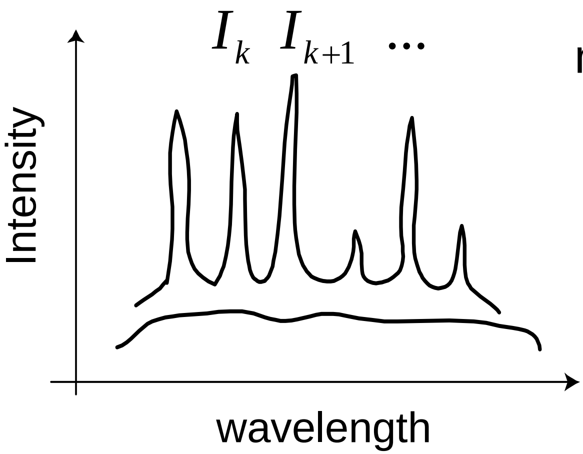
laser linewidth

$$K = \frac{\langle L | L \rangle \langle R | R \rangle}{|\langle L | R \rangle|^2} \geq 1$$

Petermann factor

PRL **89**, *083902 (2002)*:

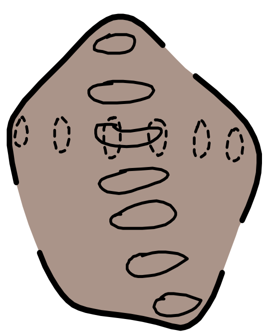
multimode lasing and mode competition



modes of passive system overlap, fast atomic medium

$$\dot{I}_k = -\kappa_k I_k + \mathcal{A}_k I_k - \sum_{k'} \mathcal{B}_{kk'} I_k I_{k'}$$

loss (↑ κ_k), linear gain (↑ $\mathcal{A}_k$), saturation (↑ $\mathcal{B}_{kk'}$)

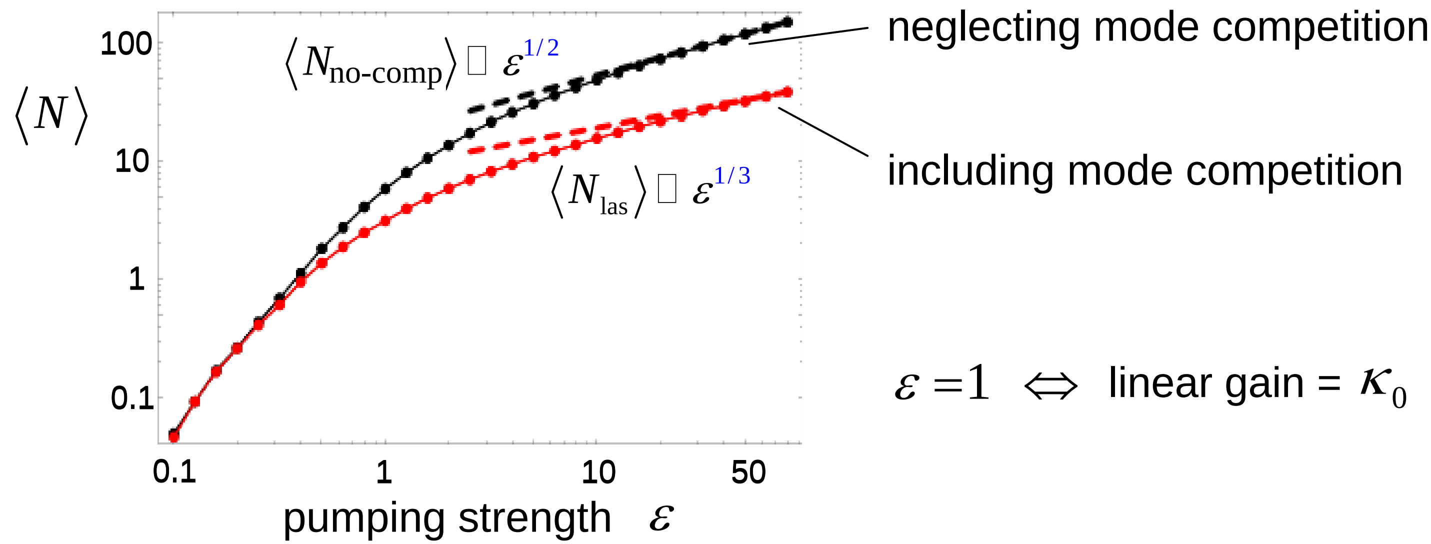


mode competition for atomic gain

$$\mathcal{A}_k \propto \int_{\text{cav}} dV\ L_k^*(\mathbf{r}) R_k(\mathbf{r})$$

$$\mathcal{B}_{kk'} \propto V_{\text{cav}} \frac{\int_{\text{cav}} dV\, L_k^*(\mathbf{r}) R_k(\mathbf{r}) R_{k'}^*(\mathbf{r}) R_{k'}(\mathbf{r})}{\int_{\text{cav}} dV\ L_k^*(\mathbf{r}) R_k(\mathbf{r}) \int_{\text{cav}} dV\ R_{k'}^*(\mathbf{r}) R_{k'}(\mathbf{r})}$$

mean number of lasing modes



$\varepsilon = 1 \Leftrightarrow$ linear gain = κ_0

- universal exponent: 1/3
- non-linear mode competition relevant

summary

- theory of (weakly confined) random lasers

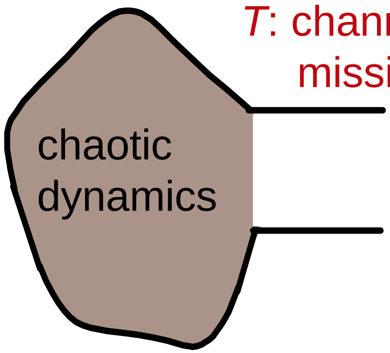
- generalization of standard laser theory

- characteristics:

  - mode coupling through damping
  - increase of laser linewidth
  - universal increase of # of lasing modes
  - increased intensity fluctuations in output (not discussed here)

thanks to: D. Savin, H. Cao

Random matrix-model



M: # channels

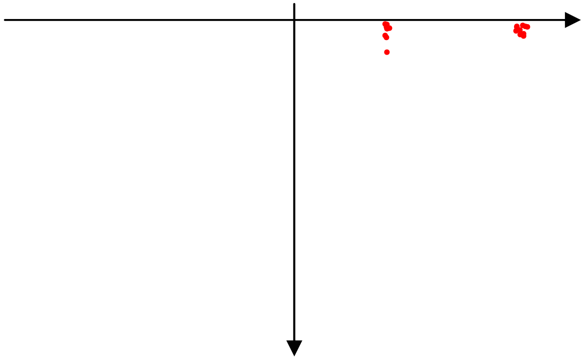
T: channel transmission coefficient

ensemble of non-Hermitian random matrices

$$\mathcal{H} = H_{\mathrm{GOE}} - i\pi W W^{\dagger}$$

internal, coupling

eigenvalues of $\mathcal{H}$: $\Omega - i\kappa$



Fyodorov & Sommers: 1997

$$\kappa_0 = \frac{TM\Delta}{4\pi}$$

frequency-separation

$$P(\kappa) \sim \begin{cases} \kappa^{M/2-1} & , \kappa \ll \kappa_0 \\ 1/\kappa^2 & , \kappa \gg \kappa_0 \end{cases}$$

➡ statistics of eigenfunctions: $\langle \mathcal{B}_{kk'} \rangle = 1 + 2\delta_{kk'}$